PHILOSOPHICAL

TRANSACTIONS

OF THE

ROYAL SOCIETY

OF

LONDON.

FOR THE YEAR MDCCCXXIX.

PART II.

LONDON:

PRINTED BY RICHARD TAYLOR, RED LION COURT, FLEET STREET.

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Dr. William Olbers.

XIX.—*Consideration of the objections raised against the geometrical representation of the square roots of negative quantities. By the Rev.* JOHN WARREN, *M.A. of Jesus College, Cambridge. Communicated by* THOMAS YOUNG, *M.D. Foreign Secretary to the Royal Society.*

Read February 19, 1829.

SOME years ago my attention was drawn to those algebraic quantities, which are commonly called impossible roots or imaginary quantities: it appeared extraordinary, that mathematicians should be able by means of these quantities to pursue their investigations, both in pure and mixed mathematics, and to arrive at results which agree with the results obtained by other independent processes; and yet that the real nature of these quantities should be entirely unknown, and even their real existence denied. One thing was evident respecting them; that they were quantities capable of undergoing algebraic operations analogous to the operations performed on what are called possible quantities, and of producing correct results: thus it was manifest, that the operations of algebra were more comprehensive than the definitions and fundamental principles; that is, that they extended to a class of quantities, viz. those commonly called impossible roots, to which the definitions and fundamental principles were inapplicable. It seemed probable, therefore, that there was a deficiency in the definitions and fundamental principles of algebra; and that other definitions and fundamental principles might be discovered of a more comprehensive nature, which would extend to every class of quantities to which the operations of algebra were applicable; that is, both to possible and impossible quantities, as they are called. I was induced therefore to

examine into the nature of algebraic operations, with a view, if possible, of arriving at these general definitions and fundamental principles: and I found, that, by considering algebra merely as applied to geometry, such principles and definitions might be obtained. The fundamental principles and definitions which I arrived at were these: that all straight lines drawn in a given plane from a given point, in any direction whatever, are capable of being algebraically represented, both in length and direction; that the addition of such lines (when estimated both in length and direction) must be performed in the same manner as composition of motion in dynamics; and that four such lines are proportionals, both in length and direction, when they are proportionals in length, and the fourth is inclined to the third at the same angle that the second is to the first. From these principles I deduced, that, if a line drawn in any given direction be assumed as a positive quantity, and consequently its opposite, a negative quantity, a line drawn at right angles to the positive or negative direction will be the square root of a negative quantity, and a line drawn in an oblique direction will be the sum of two quantities, the one either positive or negative, and the other, the square root of a negative quantity.

This may be illustrated by the following examples:

(1) Let it be required to find the length and direction of $\sqrt{-1}$;

First to find the direction of $\sqrt{-1}$,

$\sqrt{-1}$ is evidently a mean proportional between $+1$, and -1;

Now by the definition of proportion, if 4 lines be proportionals, the fourth is inclined to the third at the same angle that the second is to the first,

$\therefore$ if three lines be proportionals, the third is inclined to the second at the same angle that the second is to the first,

$\therefore$ a mean proportional between any two lines must lie in such a direction as to bisect the angle at which those lines are inclined to each other,

$\therefore \sqrt{-1}$ bisects the angle at which -1 is inclined to $+1$;

But -1 is inclined to $+1$ at $180°$,

$\therefore \sqrt{-1}$ is inclined to $+1$ at $90°$;

Next to find the length of $\sqrt{-1}$;

Since $\sqrt{-1}$ is a mean proportional between $+1$ and -1, and $+1$ and -1 are equal in length, $\sqrt{-1}$ is equal in length either to $+1$ or -1;

$\therefore \sqrt{-1}$ is a line equal in length to $+1$, and drawn at right angles to $+1$.

(2) Hence, if a be a positive quantity, a line equal in length to a and drawn at right angles to $+1$ will be equal to $a\sqrt{-1}$.

(3) Let it be required to find the length and direction of $\sqrt[4]{-1}$;

$\sqrt[4]{-1} = \sqrt{\sqrt{-1}}$, $\therefore \sqrt[4]{-1}$ is a mean proportional between $+1$ and $\sqrt{-1}$,

$\therefore \sqrt[4]{-1}$ is equal in length to $+1$;

Also it has been proved that $\sqrt{-1}$ is inclined to $+1$ at 90°,

$\therefore \sqrt[4]{-1}$ is inclined to $+1$ at 45°;

$\therefore \sqrt[4]{-1}$ is a line equal in length to $+1$, and inclined to $+1$ at 45°.

(4) As $\sqrt[4]{-1}$ is a line drawn in an oblique direction, let it be required to find an expression for it, considered as the sum of two quantities, the one either positive or negative, and the other the square root of a negative quantity.

Since $\sqrt[4]{-1}$ is equal in length to $+1$ and is inclined to $+1$ at 45°, and addition is performed in the same manner as composition of motion,

$$\sqrt[4]{-1} = \cos 45° + \sin 45° \sqrt{-1}$$
$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot \sqrt{-1}.$$

(5) To show that $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\sqrt{-1}$ is a true value of $\sqrt[4]{-1}$ according to common algebra;

Let $\sqrt[4]{-1} = x$,

then $x^4 + 1 = 0$,

an equation, one of whose roots is $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\sqrt{-1}$,

$\therefore \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\sqrt{-1}$ is a true value of $\sqrt[4]{-1}$.

In like manner other examples might be given, but these will suffice to illustrate the definition; and for further information I must refer to a treatise which I published on this subject in April 1828.

Since the publication of that work, several objections have been made to the geometrical representation of the square roots of negative quantities. First, that impossible roots are merely signs of impossibility; that, if, in the solution of any question, we arrive at an equation, all whose roots are impossible, the only conclusion to be drawn, is, that the question involves an impos-

sibility; and therefore it is absurd to suppose that the square roots of negative quantities can have any real existence. A second objection is, that there is no necessary connexion between algebra and geometry, and therefore that it is improper to introduce geometric considerations into questions purely algebraic; and that the geometric representation, if any exists, can only be analogical, and not a true algebraic representation of the roots. A third objection is, that this geometric representation, even if it be a correct representation of the roots, is merely a matter of curiosity, and can be of no use to mathematicians.—The object of this paper is to answer these objections.

To the first objection, that impossible roots are merely signs of impossibility, it may be replied, that, though they are so in some questions they are not necessarily so in all, and that in this respect they resemble fractional and negative roots. It is obvious, that in all equations derived from suppositions, which involve an impossibility or absurdity, the impossibility or absurdity will show itself in the result of the operation: and this may appear as well by a fractional root or by a negative root, as by a root commonly called impossible: thus, if we have a question, which from its nature does not admit of a fractional answer, and in resolving this question we arrive at an equation, which only admits of fractional roots, these fractional roots are in this case a proof, that the question involves an impossibility. Also if a question does not admit of a negative answer, and in resolving it we arrive at an equation which only admits of negative roots, in this case also we conclude that the question involves an impossibility. So, in like manner, if in resolving a question which does not admit of what are now commonly called impossible roots as answers, we arrive at an equation, all of whose roots are impossible, we must conclude, that the question involves an impossibility; and we have no greater reason for inferring from the last case, that what are called impossible roots have no real existence, than we have for inferring from the two former cases, that fractional or negative quantities have no real existence. This will be rendered clearer by an example: Let a body revolve in a circle by the action of a centripetal force, which varies inversely as the nth power of the distance; and let it be required to find the height from which a body must fall to the circle to acquire the velocity of the body revolving in the circle. In this example, if n be greater than 3, the velocity of the body revolving in the circle is greater than the velo-

city which can be acquired by falling from any distance however great; therefore the question involves an impossibility: therefore, if we obtain an equation for determining the height from which a body must fall to acquire a velocity equal to the velocity in the circle, the equation must, when n is greater than 3, show, in some way, that the question involves an impossibility.

Let r be the radius of the circle, and x the height (measuring from the centre) from which the body must fall to acquire the velocity in the circle; then we obtain the following equation.

$$\frac{1}{r^{n-1}} = \frac{2}{n-1} \cdot \left\{ \frac{1}{r^{n-1}} - \frac{1}{x^{n-1}} \right\}.$$

From which we deduce $x^{n-1} - \frac{2}{3-n} \cdot r^{n-1} = 0$.

Now, by the nature of the question, x must be positive; therefore whenever the above equation has no positive root, the question must involve an impossibility.

Let $n = 5$, then $x^4 + r^4 = 0$, an equation, all whose roots are what are called impossible roots; therefore, since the equation has no positive root, the question involves an impossibility.

Next, let $n = 6$, then $x^5 + \frac{2}{3} r^5 = 0$, an equation, one of whose roots is negative, and the other four what are called impossible roots; therefore in this case also, since the equation has no positive root, the question involves an impossibility.

Therefore a negative root may be a sign of impossibility, as well as what is called an impossible root.

In like manner other examples might be given, from which it would appear, that, in some cases, fractional roots may also be signs of impossibility.

Therefore we have no stronger reasons a priori to determine, that, what are called impossible roots, have no real existence, because, in some cases, they are signs of impossibility, than we have to determine that fractional or negative roots have no real existence, because, in some cases, they also are signs of impossibility.

To the second objection, viz. that there is no necessary connexion between algebra and geometry, it may be answered, that there is a connexion between what are called impossible roots and the series for the circumference of the circle, which connexion may be proved on principles purely algebraic, without the intervention of any geometric considerations.

This will appear from the expansion of 1^x;

One of the values of 1^x is 1,

This value is not a function of x,

But 1^x has other values, which are functions of x;

For example, let $x = \frac{1}{3}$, and let $1^{\frac{1}{3}} = y$,

then $y^3 - 1 = 0$,

an equation, whose roots are $1, \frac{-1+\sqrt{-3}}{2}, \frac{-1-\sqrt{-3}}{2}$;

Next, let $x = \frac{1}{4}$, and let $1^{\frac{1}{4}} = y$,

then $y^4 - 1 = 0$,

an equation whose roots are $1, -1, +\sqrt{-1}, -\sqrt{-1}$;

In like manner it will appear, if other values be given to x, that 1^x will have values which are dependent upon the values of x;

that is, 1^x is a function of x;

$\therefore 1^x$ may be expressed in the form $A + Bx + Cx^2 + \&c.$

where A, B, C, &c. are constant quantities independent of the value of x;

First, to find the value of A, Let $x = 0$, then $1^0 = A$,

But $1^0 = 1$, $\therefore A = 1$; $\therefore 1^x = 1 + Bx + Cx^2 + \&c.$;

Next, to find the law of the series, $1^x \cdot 1^y = 1^{x+y}$,

$\therefore$ the series is of the form, $1 + Bx + \frac{B^2x^2}{1.2} + \frac{B^3x^3}{1.2.3} + \&c.$;

Next, to find the value of B, $1^{nx} = 1 + Bnx + \frac{B^2n^2x^2}{1.2} + \&c.$;

Let $\frac{1^n - 1}{1^n + 1} = m$, then $1^n = \frac{1+m}{1-m}$,

$\therefore 1^{nx} = \frac{(1+m)^x}{(1-m)^x} = (1+m)^x \cdot (1-m)^{-x}$;

Let $M = m - \frac{1}{2}m^2 + \frac{1}{3}m^3 - \frac{1}{4}m^4 + \&c.$

$M' = -m - \frac{1}{2}m^2 - \frac{1}{3}m^3 - \frac{1}{4}m^4 - \&c.$

Then $(1+m)^x = 1 + Mx + \frac{M^2x^2}{1.2} + \&c.$

$(1-m)^{-x} = 1 - M'x + \frac{M'^2x^2}{1.2} - \&c.$

$\therefore 1^{nx} = \left(1 + Mx + \frac{M^2x^2}{1.2} + \&c.\right) \cdot \left(1 - M'x + \frac{M'^2x^2}{1.2} - \&c.\right)$

$$= 1 + (M - M') + x\frac{(M - M')^2 x^2}{1.2} + \&c.$$

$$\therefore 1 + Bnx + \frac{B^2 n^2 x^2}{1.2} + \&c. = 1 + (M - M')x + \frac{(M - M')^2 x^2}{1.2} + \&c.$$

$$\therefore Bn = M - M' = \left\{\begin{matrix} m - \frac{1}{3}m^3 + \frac{1}{5}m^5 - \&c. \\ + m + \frac{1}{3}m^3 + \frac{1}{5}m^5 + \&c. \end{matrix}\right\}$$

$$= 2\left\{m + \frac{m^3}{3} + \frac{m^5}{5} + \&c.\right\};$$

Let $n = \frac{1}{6}$; then $m = \frac{1^{\frac{1}{6}} - 1}{1^{\frac{1}{6}} + 1}$;

Let $1^{\frac{1}{6}} = y$; then $y^6 - 1 = 0$,

an equation, one of whose roots is $\frac{1 + \sqrt{-3}}{2}$;

∴ substituting this value for $1^{\frac{1}{6}}$,

$$m = \frac{\frac{1 + \sqrt{-3}}{2} - 1}{\frac{1 + \sqrt{-3}}{2} + 1} = \frac{-1 + \sqrt{-3}}{3 + \sqrt{-3}} = \frac{(\sqrt{3} + \sqrt{-1}).\sqrt{-1}}{3 + \sqrt{-3}}$$

$$= \frac{(\sqrt{3} + \sqrt{-1}).\sqrt{-1}}{(\sqrt{3} + \sqrt{-1}).\sqrt{3}} = \frac{\sqrt{-1}}{\sqrt{3}};$$

$$\therefore B.\tfrac{1}{6} = 2\left\{\frac{\sqrt{-1}}{\sqrt{3}} + \tfrac{1}{3}\left(\frac{\sqrt{-1}}{\sqrt{3}}\right)^3 + \tfrac{1}{5}.\left(\frac{\sqrt{-1}}{\sqrt{3}}\right)^5 + \&c.\right\},$$

$$\therefore B = 12\left\{\frac{1}{\sqrt{3}} - \tfrac{1}{3}\left(\frac{1}{\sqrt{3}}\right)^3 + \tfrac{1}{5}.\left(\frac{1}{\sqrt{3}}\right)^5 - \&c.\right\}.\sqrt{-1};$$

Now $12\left\{\frac{1}{\sqrt{3}} - \frac{1}{3}\left(\frac{1}{\sqrt{3}}\right)^3 + \frac{1}{5}\left(\frac{1}{\sqrt{3}}\right)^5 - \&c.\right\}$ is a series, which expresses the value of the circumference of a circle, whose radius is unity;

Let this series $= c$, then $B = c\sqrt{-1}$,

$$\therefore 1^x = 1 + cx\sqrt{-1} - \frac{c^2 x^2}{1.2} - \frac{c^3 x^3}{1.2.3}\sqrt{-1} + \frac{c^4 x^4}{1.2.3.4} + \&c.;$$

∴ We have by means of mere algebraical operations, without the introduction of any geometrical considerations, expanded 1^x in a series, which involves c, the circumference of a circle whose radius is unity.

And this series was obtained by substituting for 1^n one of its impossible values as they are called.

∴ There is a connexion between what are called impossible roots and the circumference of the circle.

But by examining the series more accurately, we may find a greater connexion between the series and the properties of the circle

$$\text{For } 1 - \frac{c^2x^2}{1.2} + \frac{c^4x^4}{1.2.3.4} - \&c. = \cos cx,$$

$$\text{and } cx - \frac{c^3x^3}{1.2.3} + \frac{c^5x^5}{1.2.3.4.5} - \&c. = \sin cx,$$

$$\therefore 1^x = \cos cx + \sin cx \cdot \sqrt{-1}.$$

Now lest there should be any error in the method of expanding 1^x as given above, let us try whether we can verify the last equation by some independent process;

Let $x = \frac{p}{q}$, where p and q are whole numbers,

$$\text{and let } \overset{\frac{1}{q}}{1} = y \text{ then } y^q = 1,$$

an equation, one of whose roots is $\cos \frac{c}{q} + \sin \frac{c}{p} \sqrt{-1}$, where c is the circumference of a circle, whose radius is unity,

$$\therefore \overset{\frac{1}{q}}{1} = \cos \frac{c}{q} + \sin \frac{c}{q} \cdot \sqrt{-1},$$

$$\therefore \overset{\frac{p}{q}}{1} = \left(\cos \frac{c}{q} + \sin \frac{c}{q} \cdot \sqrt{-1}\right)^p = \cos \frac{p}{q} c + \sin \frac{p}{q} c \cdot \sqrt{-1},$$

$$\text{or } 1^x = \cos cx + \sin cx \cdot \sqrt{-1},$$

the same equation as that obtained above by the expansion of 1^x.

From what has just been proved, it appears that there is a connexion between the properties of the circle and the quantities commonly called impossible roots, that is between geometry and algebra; therefore it is so far from being improper to introduce geometric considerations into questions purely algebraic, that it is to geometry we must look (and to geometry alone as far as we know at present), if we expect to arrive at a true theory respecting the square roots of negative quantities.

It may be proper to observe here, that the object of the above investigation is not to expand all the values of 1^x in a series, but merely to show that one

of them can be so expanded without the intervention of any geometric considerations, and to point out that the series obtained will involve c the circumference of the circle, and thus to prove that there is a connexion between algebra and geometry; if the object had been to obtain all the values of 1^r in a series, it would have been more convenient to have introduced geometrical considerations as in my treatise.

To the second part of this objection, viz. that the geometric representation can only be analogical and not a true algebraic representation of the roots; it may be replied, that the geometric representation of the square roots of negative quantities rests on the same foundation as the geometric representation, or any other representation of the negative quantities themselves. A negative quantity arithmetically considered is a mere absurdity, being the difference which arises from subtracting a greater quantity from a less; but algebraists having found that operations might more easily be performed by considering negative quantities in the abstract, endeavoured to establish their real existence, and with this view they made the following hypotheses; that, if a line drawn in one direction be represented by a positive quantity, a line drawn in the opposite direction will be represented by a negative quantity; that the sum of a positive and a negative quantity is to be found by subtracting the less from the greater, and prefixing the sign of the greater; that subtraction is to be performed by changing the sign of the quantity to be subtracted, and proceeding as in addition; that the product of a positive and a negative quantity is negative, and the product of two negative quantities, positive; and having made these hypotheses, they proved, by examining into the nature of algebraic operations, that the results arrived at by means of these hypotheses must be correct; therefore they concluded that these were true hypotheses; and their truth being established, they were admitted as fundamental principles of algebra: and in the same way other true hypotheses were established relative to the representation of negative quantities: such as; if time to come be represented by a positive quantity, time past will be represented by a negative quantity, &c. I call these algebraic principles, hypotheses; for though most algebraists have considered them as propositions, and have endeavoured to establish their truth by direct demonstration, yet their reasoning is unsatisfactory, for they always treat of negative quantities

as quantities to be subtracted, therefore their proofs are only applicable to the difference of two positive quantities, and not to negative quantities abstractedly considered. These fundamental principles must therefore be looked upon as hypotheses introduced into algebra in order to give to negative quantities a representation and a real existence. And in like manner, in order to arrive at the representation of the square roots of negative quantities, I have made the following hypotheses: that all straight lines drawn in a given plane from a given point in any direction whatever, may be algebraically represented both in length and direction: that addition is performed in the same manner as composition of motion in dynamics; that four straight lines are proportionals, both in length and direction, when they are proportionals in length, and the fourth is inclined to the third at the same angle at which the second is inclined to the first: and I have by means of these hypotheses as a foundation, established all the common rules for performing algebraic operations, and thus have proved, that the results arrived at by means of these hypotheses must be correct: therefore I conclude, that these are true hypotheses, and true in the same sense, that the hypotheses made by algebraists respecting the representation of negative quantities are true. In fact, if there be a question, whether negative quantities can or cannot be represented geometrically; the only way in which such a question can be solved, is by making certain hypotheses with respect to their geometric representation, and then showing that the results arrived at from these hypotheses must be correct: and in like manner if there be a question whether those quantities commonly called impossible can be geometrically represented, the question must be solved in the same way; viz. by making certain hypotheses respecting them, and showing that the results arrived at by means of these hypotheses must be correct. In this point of view, the definitions and fundamental principles which I have laid down in my treatise must be considered as mere hypotheses; and mathematicians will be satisfied of their correctness when they see that the results agree in every respect with the results obtained by other independent processes.

To the third objection, viz. that the geometric representation of the square roots of negative quantities can be of no use to mathematicians, it will not be necessary to say much in reply.

In the works which have lately been written, either on pure or mixed mathe-

matics, we may observe that great use is made of impossible roots; and we may fairly conclude that if these quantities are of so great service to mathematicians, even while they are ignorant of their real nature, they will be of much greater service when the true theory respecting them is known; we may reasonably expect, that our knowledge of algebra will be increased when the nature of impossible roots is understood in the same manner as that of possible roots; these are the general advantages which we shall derive from the geometric representation of the square roots of negative quantities: but there is one particular advantage, and that, one of the greatest importance, which arises from the definition of addition; addition is performed in the same manner as composition of motion in dynamics, therefore any question in dynamics where the motion of the bodies is confined to one plane, becomes a mere question of algebra, the laws of motion being contained in the definitions of algebra.

Before I conclude this paper, it will be proper to take notice of two works which have appeared on this subject; the first a paper in the Philosophical Transactions, for the year 1806, p. 23: intitled "Mémoire sur les Quantités Imaginaires, par M. Bué;" the second, a work intitled "La vraie Théorie des Quantités Négatives et des Quantités prétendues Imaginaires, par C. V. Mourey, Paris, 1828." I was not aware of the existence of M. Bué's paper till November 1827, when my treatise was in the press: at that time I read his paper, and also the article upon it in the Edinburgh Review of July 1808. M. Bué begins with stating that the negative sign has two different significations in algebra; viz. that if algebra be considered as a universal arithmetic, the negative sign is a sign of subtraction, but that if algebra be considered as a mathematical language, the negative sign is a sign of a quality; on this point he makes the following observations:

"Considérés comme signes d'opérations arithmétiques, + et − sont les signes, l'un de l'addition, l'autre de la soustraction."

"Considérés comme signes d'opérations géométriques, ils indiquent des directions opposées. Si l'un, par exemple, signifie qu'une ligne doit être tirée de gauche à droite, l'autre signifie qu'elle doit être tirée de droite à gauche."....

"Mis devant une quantité, q, ils peuvent désigner, comme je l'ai dit, deux opérations arithmétiques opposées dont cette quantité est le sujet. Devant

cette même quantité, ils peuvent désigner deux qualités opposées ayant pour sujet les unités dont cette quantité est composée.

"Dans l'algèbre ordinaire, c'est à dire, dans l'algèbre considérée comme arithmétique universelle, ou l'on fait abstraction de toute espèce de qualité, les signes + et − ne peuvent avoir que la première de ces significations". . . . toutes les fois qu'on a pour résultat d'une opération une quantité précédée du signe −, il faut, pourque ce résultat ait un sens, y considérer quelque qualité. Alors l'algèbre ne doit plus être regardée simplement comme une arithmétique universelle, mais comme une langue mathématique."

He then proceeds to the sign $\sqrt{-1}$: this he considers a sign of perpendicularity; he argues that it is a mean proportional between + 1 and − 1, and therefore must be a perpendicular; he also gives another proof that it is a perpendicular; he makes a square to revolve through 90° about one of its angular points, and observes, that if the square is positive in its first situation, it will be negative after having moved through 90°; therefore if the square in its first situation be represented by + 1, it will in its second situation be represented by − 1, and its side will in the first case be represented by + 1 or − 1, and in the second by $+\sqrt{-1}$ or $-\sqrt{-1}$; but the side of the square has moved through 90°; therefore he concludes that $\sqrt{-1}$ is a sign of perpendicularity. In the above demonstration M. Buée applies his method of reasoning as well to areas as to lines; but as in my treatise I have confined myself to the algebraical representation of lines, I will not make any observation respecting the force of this proof. M. Buée afterwards proceeds to say, that though perpendicularity is properly the only quality indicated by $\sqrt{-1}$, yet $\sqrt{-1}$ may be made to signify any other quality, provided we can reason respecting that quality in the same manner as we reason respecting perpendicularity; he then gives examples illustrative of his theory: some of these examples I cannot understand; others are more clear; but in almost all there is one great defect, viz. he is obliged to introduce some arbitrary limitation into the question, in order to make the answer agree with the root of the equation: this arises from the want of a general geometrical definition of proportion or multiplication, which is necessary to render the theory complete: he also endeavours to prove that $(\sqrt{-1})^n = n\sqrt{-1}$; but this I cannot comprehend. However, notwithstanding these defects or errors, the general principles on which he reasons

are good; he evidently proceeds on the principle, that whenever in the algebraic solution of any question, we arrive at imaginary quantities as answers, we must consider that the question might have been expressed in more general terms, and that the imaginary quantities are answers to the question in this extended sense. This appears to me to be the true principle, and is analogous to our usual method of reasoning, when we arrive at a negative answer in resolving a question, which, from the manner in which it is expressed, only admits of positive answers.

The Edinburgh reviewers in their article on M. Buée's "mémoire," state their opinion with respect to the nature of the square roots of negative quantities in these words:

"The essential character of imaginary expressions is to denote impossibility; and nothing can deprive them of this signification, nothing like a geometrical construction can be applied to them; they are indications of the impossibility of any such construction, or of any thing that can be exhibited to the senses."

As I have already answered this objection, it will not be necessary for me to make any further remarks on this point.

In considering the evidence adduced by M. Buée in support of his fundamental proposition, that $\sqrt{-1}$ expresses perpendicularity, the reviewers begin with giving his reasoning on that subject, viz. $\sqrt{-1}$ is a mean proportional between $+1$ and -1, and therefore a perpendicular; and they observe with respect to his arguments, that "any imaginable conclusion might have been obtained in the same manner, the third line for example, needed not have been placed at right angles to the other two, but making an angle, suppose of 120° with one, and of 60° with the other; it would still be a mean proportional between them, and its square would be therefore, according to the above method of reasoning equal to $+1 \times -1 = -1$, so that the line itself would be equal to $\sqrt{-1}$, and thus $\sqrt{-1}$ would denote not perpendicularity, or the situation in which a line makes the adjacent angles equal, but that in which it makes one of these angles double of the other; the one of these arguments is just as good as the other, and neither of them of course is of any value."

The above objection derives its force from the want of a definition of proportion in M. Buée's "mémoire," as is evident from what has already been proved in this paper.

I saw M. Mourey's work in December 1828, and found that his method of considering the subject is nearly the same as the method which I have adopted in my treatise: but he has in his work a proof that every equation has as many roots as it has dimensions, which I have not in mine; this proof with a very slight alteration I communicated to the Philosophical Society at Cambridge. My reason for introducing an alteration was this: the author, after having taken (in the figure which he makes use of) as many points as the given equation has dimensions, and proved that round each point there is a curve which has certain properties, and that in each curve there is a line which will satisfy the conditions of the equation, concludes that there are as many lines which will satisfy the conditions of the equation as the equation has dimensions; which conclusion does not necessarily follow from the premises; for one curve may surround two or more of the points in his figure, in which case he ought to have proved, that if any one of the curves surrounds m of the points, there will be m lines in that curve, which satisfy the conditions required, which he has not done, therefore his proof is in that part defective; consequently an alteration was necessary; and the alteration was easily made, as it is enough to prove, that an equation of n dimensions has one root, after which it may be depressed to an equation of $n-1$ dimensions. In all other respects the proof given by M. Mourey is remarkably clear and satisfactory, and an example of the advantages which mathematicians may derive from a knowledge of the true theory of the quantities improperly called impossible or imaginary.

XX. *Anatomical description of the foot of a Chinese female. By* BRANSBY BLAKE COOPER, *Esq., Surgeon to Guy's Hospital. Communicated by* PETER MARK ROGET, *M.D., Secretary to the Royal Society.*

Read March 5, 1829.

A SPECIMEN of a Chinese foot, the account of which I have the honour to lay before the Royal Society, was removed from the dead body of a female found floating in the river at Canton. On its arrival in England it was presented to Sir ASTLEY COOPER, to whose kindness I am indebted for the opportunity of making this curious dissection. Without entering into an inquiry whether this singular construction, and as we should esteem it hideous deformity of the Chinese female foot, had its origin in Oriental jealousy, or was the result of an unnatural taste in beauty; I shall content myself with describing the remarkable deviations from original structure, which it almost every where presents. It may be proper, however, to remark, that as this conformation is the result of art, commenced at the earliest age and exercised on the persons of females only, we should naturally expect to find the most perfect specimens among those of the highest rank. Now as this body was found under circumstances which lead me to suppose that it was one of the lower orders, the measured proportions of the foot are therefore to be considered somewhat above the more successful results of this cruel art when completed on the feet of those in more exalted stations of life.

To an unpractised eye, the Chinese foot has more the appearance of a congenital malformation than the effect of art, however long continued; and although no real luxation has taken place, yet at first sight we should either consider it as that species of deformity vulgarly called club-foot, or the result of some accidental dislocation, which from ignorance and want of surgical skill, had been left unreduced.

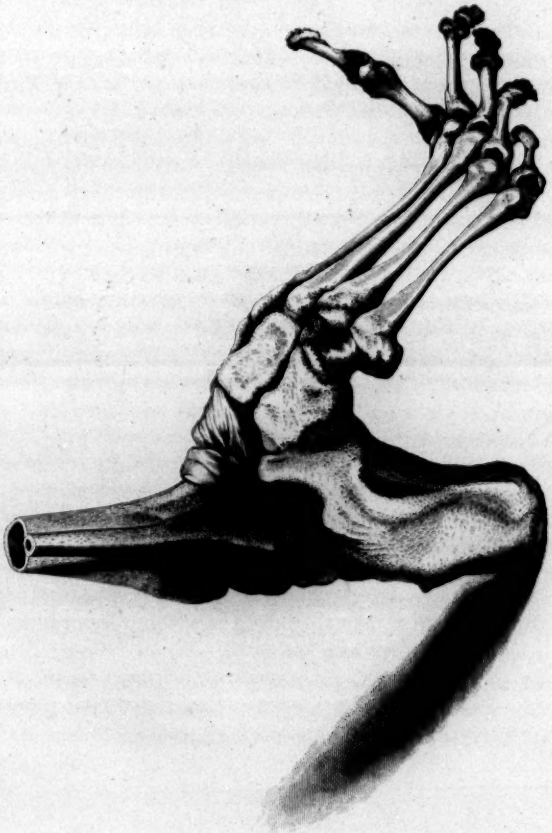
From the diminutive size of the foot, the height of the instep, the want of

breadth, and above all, the extremely dense nature of the cellular tissue of the foot, it is evident that progression must at all times be difficult; and even the poising of the body when in the erect position, must require unusual exertion of muscular power, which, considering the disadvantages with which these muscles have to contend, is a matter of no small astonishment.

From the heel to the great toe the foot is unusually short, not exceeding five inches, and is said in some instances to measure even less than this; and the great toe itself, which in its natural and free state projects in a straight forward direction, is bent with a peculiar abruptness upwards and backwards, whilst the remaining toes, with the exception of the first phalanx of the second and third, are doubled in beneath the sole of the foot, so as to leave scarcely any breadth at this part of the foot, which in the unconstrained limb is commonly the broadest; and the striking shortness of the heel scarcely projecting beyond the line of the leg, which itself descends upon the foot at a considerable obliquity from behind forwards, imparts an appearance to the foot, as if it were kept in a state of permanent extension. The upper surface of the foot is very convex; but its convexity is irregular and unnatural, presenting a sudden and prominent projection just anterior to the external malleolus, and above the outer extremity of a deep cleft which traverses the sole of the foot. But as it is in the sole that the most remarkable alterations are produced, I shall give a particular description of it first.

Sole of the Foot.

In describing the sole, we will suppose the foot to rest upon the heel, as it would do were the individual placed horizontally upon the back. In this view we observe the great toe bent backwards towards the leg, and immediately beneath the articulation of its two phalanges the second toe is so twisted under it that its extremity reaches to the inner edge of the foot; its nail occupies the centre of this position, having a considerable projection of integument beyond it. Next, but still anterior to the ball of the great toe, are the two extreme phalanges of the third toe; they are placed more obliquely than the phalanges of the second toe, and consequently do not reach so far inwards across the foot. The nail of this toe is somewhat nearer its extremity, but more completely on its anterior surface, so as nearly to touch the edge of the pre-



F. Bauer, del.

ceding one. A corn which appears on the space external and posterior to the nail of this toe, seems to indicate that, as the point of the fore part of the foot which is first subjected to pressure. We come now to the ball of the great toe, which separates the toes already described, from the two outer ones: it does not present its usual full convex appearance, but is flattened on its under surface, and compressed from before backwards by the position of the third and fourth toes. The position of the two remaining toes is very remarkable, and differs essentially from that of the others: for while in them only two phalanges are bent under the plantar region of the foot, in these all the phalanges are doubled beneath it in such a manner as to produce a visible depression in the external edge of the foot. The fourth toe is placed more obliquely than the third, with its nail very much contracted, and is situated on its anterior edge; a large corn presents itself more external to the nail than in the third toe. The last or fifth toe stretches in a transverse direction across the under surface of the foot, and forms the anterior boundary to a deep cleft which occupies the centre of the sole. This toe is so much expanded as to appear the largest; externally and posterior to its nail it has two corns, placed much in the same manner as that on the fourth toe. But the strangest feature in this deformity is the cleft, or hollow just mentioned; it is very deep, with a slight obliquity from without inwards, and extends transversely across the whole breadth of the foot between the toes and the heel. To judge from its appearance, one might suppose that the heel and toes had been forcibly brought together, so as considerably to diminish the whole length of the foot, and to convert its natural longitudinal hollow into that deep concavity. The heel which forms the other boundary of the cleft, presents a large square surface, if not entirely flattened, yet with a striking diminution of convexity, so as to suggest the probability that it affords the principal point of support in progression; a surmise which is further corroborated by the great density of the skin of this part.

Dorsum of the Foot.

The external character of the foot is completely altered here also; the direction of the leg downward and forward, forming before an obtuse angle with the foot, so as to give it an appearance of permanent extension, is the first cir-

cumstance worthy of notice. The dorsum rises with an unusual convexity, not only from behind forward, but also from side to side: it affords a distinct protuberance situated just before the external malleolus, and above the outer extremity of the cleft in the sole, which is here very conspicuous; anterior to this eminence, the dorsum presents a plane surface facing outwards, till it slopes off rapidly beneath where the toes are turned under the sole. There is but a trifling alteration in the aspect of the inner surface of the dorsum; this side of the foot having undergone but little distortion: but the manner in which the dorsum is united with the great toe deserves yet to be particularly noticed. A considerable angle distinguishes their point of junction, resulting from the dent or hollow, which the abrupt direction of the great toe upwards and forwards produces upon that surface. In this view we have the dorsum of the great toe with its aspect directly upwards; whilst the inner surface of the first phalanx of the second toe has its dorsum turned outwards. Only a small portion of the inner surface of the third toe can be perceived in this view, whilst the remaining toes are buried beneath the foot. Posteriorly there is little to remark, beyond the extreme shortness of the heel, which is not flatter, but wider than in the natural condition.

The integuments covering the heel are unusually dense, hard, and resisting, and the cuticle is of a remarkable thickness. The subcutaneous structure resembles rather the fatty sole of a horse's foot than any human tissue. The skin which covers the rest of the sole presents a corrugated appearance, and is somewhat thicker than in an ordinary foot: but in those places where it had been defended from external pressure by the intervention of the toes, which passed under it, it does not deviate from the natural construction.

On the dorsum, the integuments offer nothing unusual: unless it be that the nail of the great toe, as might be anticipated from constant compression, is rendered particularly convex from side to side.

The other nails are not visible in this aspect of the foot.

The tendons do not appear to have undergone any change, further than as their direction depended upon the altered position of the bones.

It is however in the skeleton of the foot that we observe the greatest changes produced by art. The powerful effect of long continued pressure over the direction even of the bones is here very striking.

The position of the os calcis is very remarkably altered: instead of the posterior projection which usually forms the heel, a straight line is preserved in this direction, not deviating from the line of the tibia; and the projecting point which forms in an ordinary foot the most posterior process, and into which the tendo Achillis is inserted, touches the ground and becomes the point d'appui for sustaining the whole weight of the body. The articular surface of the calcis, in connection with the cuboid bone, is about half an inch anterior to, and two inches above this point; while the astragalar joint is behind, and somewhat below, the calco-cuboidal articulation; consequently the direction of the os calcis (in its long axis) instead of being from behind forwards, is from below upwards, with the slightest possible inclination forwards. The most prominent parts of the instep are the round head of the astragalus, and the cuboidal articulation of the os calcis. From this the remaining tarsal bones slope downwards at nearly a right-angular inclination to join the metatarsal bones, whose obliquity is still downwards, until they rest on their phalangeal extremities.

The length between the os calcis where it touches the ground, and the most anterior part of the metatarsal bone of the great toe, is 4 inches.

The length of the foot including the toes $5\frac{1}{4}$ inches.

The height of the instep $3\frac{1}{2}$ inches.

Thus the arch of the foot has a span of two inches and a quarter with the height of two inches, which space is filled up with the condensed cellular substance before described.

The cleft of the sole traverses the foot at this place, and is three inches in depth. The width of the foot at its broadest part is barely two inches.

The points of support are the os calcis, the anterior extremity of the metatarsal bone of the great toe, and the dorsal surface of the fourth and fifth toes, which are bent under the foot so as to press the ground at this part.

Such are the anatomical particulars of this singular deformity; and though Nature has, by providing an accumulation of fat, thickening the skin and cuticle, and widening the surface of the heel, done her utmost to rectify the evil consequences of an unnatural custom, yet the awkward gait of a person attempting to walk on such deformed members may be easily imagined. Under such circumstances, in order to preserve equilibrium in an attempt to walk, it must be necessary to bend the body forwards in an uneasy position,

and at the expense of a muscular exertion, which in ordinary progression is not put forth. To what extent the general health of the unfortunate individual thus deprived of the natural means of exertion may be affected, is a curious subject of inquiry, and remains I believe to be ascertained.

I may be permitted to add, that the existence of this extraordinary custom, though familiar to our ears, is presented in a forcible light to our imagination by such a specimen as I have the honour to present to the Royal Society.

In offering to the Royal Society this brief sketch of the dissected foot, I do not pretend to attach to the subject any more importance than it deserves; nevertheless I have thought it would be considered as curious, and calculated to interest scientific men. And further, as its description has hitherto formed a desideratum in our accounts of anatomical curiosities, I have thought that my endeavour to supply it would not be unacceptable.

XXI. *Some observations on the functions of the nervous system, and the relation which they bear to the other vital functions.* By ALEXANDER PHILIP WILSON PHILIP, *M.D. F.R.S. L. & E.*

Read April 2, 1829.

THE experiments relating to the function of digestion detailed or referred to in a paper which I lately had the honour to present to the Society, appear to throw light on the function of the ganglionic nerves, which hold a higher place in the animal economy than those either of sensation merely or voluntary power, being as essentially a vital organ as the heart or lungs, as will more fully appear, I think, from the review of facts which I now beg leave to submit to the Society.

For the last fifteen years I have been engaged in an experimental inquiry relating to the laws of the vital functions; and have from time to time laid the results before the Royal Society in six papers, which the Society has done me the honour to publish. All the experiments on which the statements are founded, having been made in the presence of competent witnesses, the rule from which I never deviated, has been to repeat each experiment till no doubt respecting the result remained in the mind of any one present; and it is satisfactory to me to be enabled to state, that, although many of these experiments have been repeated by the physiologists both of this country and the continent, they have in no instance been found inaccurate. I have always abstained from troubling the Society till I had some new facts to state, which appeared to me to deserve its attention; and I have confined myself to the simple statement of the facts and the means by which they were ascertained.

The present paper is offered to the Society on a different principle. It contains no new fact, but a review of what appears to me the necessary inferences from the various facts which I have had the honour to lay before it; and when the Society considers that the value of facts depends on the inferences they afford, and that the inquirer, both from his more perfect knowledge of the cir-

cumstances, and from his mind having been more particularly directed to the subject, is in several respects better fitted than others for reviewing the inferences, he hopes the following observations will not be unacceptable; especially as they are such as would naturally have made part of my former papers, had it not appeared to me better to confine myself to a simple statement of the facts, till the whole had been laid before the Society.

The present paper is offered to it for the purpose of supplying what may be regarded as a defect in those papers, and also as the conclusion of the Inquiry in which I have been so long engaged. I am fully sensible of the vast extent of the subject, and that it is only the great outline which I have attempted to trace. If this has been accurately laid down, my object has been accomplished.

The nerves may be divided into two classes, those which proceed directly from the brain and spinal marrow to the parts to which they convey the influence of these organs; and those which enter such ganglions as receive nerves proceeding from different parts of the brain and spinal marrow, whether these nerves have or have not protuberances belonging to themselves which have also been termed ganglions, but which receive only the different fibres that belong to the particular nerve to which they are attached, and from the circumstances in which they are placed, must have a different or at least a more confined relation to other parts of the nervous system. To the former, therefore, I shall for the sake of distinction, and to avoid circumlocution, confine the term ganglion.

I beg leave to lay before the Society the following extract from lectures delivered by Mr. Brodie before the College of Surgeons, and which have not yet been published, in which this accurate anatomist and physiologist has given the sum of our knowledge respecting the structure of the ganglions. "Those bodies which are found in certain nerves which appear to be formed by an enlargement of the nervous substance, and which are denominated ganglia, are of a complicated structure. Into ganglia the nervous fibres may be traced, and from these ganglia the nervous fibres again emerge. Scarpa has paid much attention to the fabric of the ganglia, and he gives the following history of it. He says that the fasciculi of nervous filaments which enter a ganglion are separated and divided from each other, and that they are combined anew. A nervous fasciculus entering a ganglion divides into smaller

fasciculi. These divide again, and cross and intersect each other at various angles. Then the divided fasciculi become again united, and as at first they divided into smaller and smaller fibres; so when they begin to unite they form gradually larger and larger bundles. At last the nerve which entered a ganglion emerges from it with its fibres collected into one or more fasciculi. Sometimes several nerves enter a ganglion, in which case they are all blended together, forming a complicated net-work, in which it is impossible to determine what belongs to one nerve and what belongs to another nerve. Every fasciculus or filament which enters a ganglion passes through it. There is no appearance of any one terminating in it."

"If we unravel the texture of a ganglion, we find that each nervous fibre retains its own peculiar neurilema; but besides this, the spaces left between the intersection of the fibres are filled up with a peculiar soft substance of a grayish or yellowish colour. With the nature of this substance we are unacquainted. Some have considered it as corresponding to the cineritious substance of the brain and spinal marrow; but SCARPA is disposed to regard it as a soft cellular substance, filled with a grayish and mucilaginous matter in emaciated subjects, and with a yellowish oily matter in those that are fat."

Such then is the structure of the ganglions as far as it is known; and as, for the reason just mentioned, I shall confine the term to those ganglions which receive nerves proceeding from different parts of the nervous system; the term ganglionic nerve I shall confine to those nerves which either enter or proceed from such ganglions, without adverting to their having or not having protuberances resembling ganglions belonging to themselves; although it is probable that a more perfect knowledge of the nervous system will point out this circumstance as a proper basis for a subdivision. It is necessary to keep this explanation in view, because neither the term ganglion nor ganglionic nerve has been employed with much precision.

Physiology has been greatly indebted to Mr. BELL for his important discovery of the different properties of the two sets of nerves which unite in forming each of the spinal nerves. It appears from his experiments, which have been confirmed by those of MAJENDIE, that the one set are nerves of sensation, the other of motion; a circumstance which explains many of the phenomena of disease, which have suggested the probability of these functions being

exercised by different nerves bound up in the same envelope. Dr. PARRY in his treatise on the pulse for example, relates a case where feeling alone was lost in one arm, and voluntary power alone in the other. But these are not the only nor indeed the most important functions of the spinal nerves. All of them contribute to the formation of the ganglionic system, on which the life of the animal, as will appear from many facts I am about to state, immediately depends.

It is evident from what has been said, that the ganglions and plexuses resemble each other in their nature; and as the nerves which terminate in them come from all the most distant parts of the nervous system, some from the brain, and some from the lower extremity, and all intermediate parts of the spinal marrow, we cannot help supposing, that there is some design in thus uniting nerves which arise from so many different parts of these organs. One of the most striking differences between the ganglionic nerves, and those proceeding directly from the brain and spinal marrow, is that even independently of the ganglions and plexuses, the former every where more freely anastomose, if I may borrow a term from the sanguiferous system; while the latter proceed in a more direct course, being less connected with each other in their progress to the parts on which they bestow sensation and voluntary power; still further demonstrating the care with which nature blends the power of the ganglionic nerves.

What purpose is served by this perpetual intertwining of these nerves? It is impossible for a moment to conceive that it is without an object. This question is most likely to be answered by inquiring into the nature and functions of the parts supplied by this class of nerves; those parts are the vital organs, the thoracic and abdominal viscera, and the vessels even, as we shall find by experiment where the parts are too minute to be made the subject of dissection, to their smallest ramifications.

It would appear from this arrangement, that, although to other parts the influence of only one part of the brain or spinal marrow is sent, the vital organs receive that of every part of them; and this inference has been confirmed by numerous experiments too simple to admit of our being deceived, which I made many years ago, and the results of which were laid before the Royal Society, and published in the Philosophical Transactions of 1815, and which

are more fully detailed in my treatise on the Vital Functions. From them it appears that although the muscles of voluntary motion obey a stimulus applied to no part of the brain and spinal marrow but that from which their nerves take their origin; the heart is influenced by stimuli applied to every part of these organs, from the very uppermost surface of the brain and cerebellum to the lowest portion of the spinal marrow. The same was found to be the case with the blood-vessels to their minutest ramifications. Even the extremities of the arteries and veins, where they unite to complete the circulation, it was found by the aid of the microscope, could be influenced, nay even deprived of power by agents whose operation was confined either to the brain or spinal marrow.

In some animals even of warm blood, as appears from experiments related in my treatise on the Vital Functions, the motion of the blood in the capillaries may be observed for an hour or even two hours after death, provided neither great and sudden injury to the nervous system, nor great loss of blood be occasioned by the mode of death; that is long after the heart has ceased to beat. The continued action of the capillaries appears from what is said in that treatise, to be the cause of the large arteries being found empty some hours after death.

It has also been shown by experiments detailed in the same treatise, an account of some of which has appeared in the Philosophical Transactions, that the stomach and lungs are in like manner under the influence of both the brain and spinal marrow.

The partial connection with the nervous system of the organs supplied by the cerebral and spinal nerves, and the universal connection with that system of those supplied by the ganglionic nerves, explain many of the phenomena, both of health and disease. Why are affections of the stomach and other vital organs felt instantly through every part of the frame, while the effects of those of a muscle of voluntary motion, or even an organ of sense, although often a part of greater sensibility, is confined to the injured part? If the eye or ear, or the muscle of a limb, be so deranged by a sudden blow, for example, as instantly to destroy its power, sight, hearing, or the voluntary power of the part is lost, and there the evil ends unless inflammation ensues; but a blow on the stomach, which instantly destroys its power, at the same moment destroys that of every other part. It is not difficult to answer the question, since the

state of the stomach, from the cause just pointed out, may influence every part of the nervous system; and it appears from experiments which the Society did me the honour to publish many years ago, some of which were repeated by Mr. Cliff, that a powerful and sudden affection of the nervous system is capable of immediately destroying the circulation in every part of the animal, by instantly depriving both the heart and blood-vessels of their power.

Here the question naturally arises. For what purpose are the vital organs thus connected with every part of the brain and spinal marrow?

This question is answered by experiments detailed in my treatise on the Vital Functions, an account of some of which appeared in the Philosophical Transactions of 1822. From them it was found that the power of secreting surfaces is deranged by abstracting from them any considerable part of the influence either of the brain or spinal marrow; and as the function of secretion is effected by the action of the nerves on the blood, as appears from facts detailed in the paper just referred to, and another which I had the honour to lay before the Society a few weeks ago, it is evident that the presence of nervous power in a secreting organ would be useless, were not the blood on which it operates also supplied, and disordered if it were not supplied in due proportion; and consequently its supply varied as the supply of nervous power varies.

We thus see not only why secreting surfaces are placed under the influence of every part of the nervous system, but also why it is necessary that the sanguiferous system should be under the controul of the same laws which regulate the supply of nervous power.

It appears then that by means of the system of ganglionic nerves, the influence of every part of the brain and spinal marrow is bestowed on secreting surfaces, and on those organs by which the supply of their fluids is regulated, and that this influence is necessary to their functions. But it is not the secreting power alone that is thus placed under the influence of every part of the brain and spinal marrow; for it is a necessary inference from experiments related in a paper which the Society did me the honour to publish last year, that the whole of those processes on which the healthy structure of the part depends are under the same influence.

The influence therefore of the whole brain and spinal marrow is thus united by nerves from various parts of these organs entering ganglions and plexuses,

from which are sent to every part of the body nerves proved by direct experiment to convey the influence of every part of them; and this combined influence of the brain and spinal marrow is employed in forming the various secreted fluids, and supporting the other processes on which the due structure of every part depends; and I have in a treatise entitled "On Indigestion" pointed out how extensively the phenomena and treatment of all diseases are influenced by this cause.

Such then is the relation which subsists between the nervous system and the other vital organs I have had occasion to mention; but there is another relation of that system which must be considered before the nature of its functions can be clearly understood.

The nervous system, in the usual acceptation of the term, is very ill defined, and functions of the most dissimilar nature are classed together under the general denomination of nervous. Those of sensation and volition, for example, are classed with the excitement of a muscle and the formation of a secreted fluid. It seems highly improbable that results so different should arise from the same or similar causes. On the most cursory view of the subject, we cannot help supposing that the nervous system, according to the common acceptation of the term, includes more than one principle of action. We have every reason to believe, that the sensorial is a power wholly distinct from that strictly called nervous; and all doubt seems to be removed by the circumstance, that although the organs of both belong to the nervous system, it is evident they are not the same organs, because the sensorial power resides chiefly in the brain while the nervous power, properly so called, resides equally in the brain and spinal marrow; the latter of which organs is capable of its functions independently of the former, as appears from many of the experiments of Le Gallois, which have been confirmed by several of my own.

It occurred to me on reviewing the whole of these circumstances, that as we can destroy the nervous, without at all impairing the muscular power, it might be possible to remove the sensorial power without immediately destroying that more strictly called nervous.

I made many experiments, which are detailed in my treatise on the Vital Functions, for the purpose of determining this point; from which it appears that in all modes of death, except the most sudden, (arising from a violent and

sudden impression made on the nervous system, by which the whole of the functions are instantaneously destroyed,) the sensorial functions are the first which cease, all the other powers of the system remaining more or less perfect, and any imperfection which appears in them not directly depending on the loss of the sensorial power.

Of the sensorial functions, sensation and volition are the only ones which we are called upon to consider here, because they alone have any share in maintaining animal life. That these functions are essential to the maintenance of life in all the more perfect animals, will, I think, appear from what I am about to lay before the Society.

The following may be regarded as the nervous functions properly so called. The excitement of the muscles of voluntary motion, by which through the intervention of the nervous system, they in their usual functions are subjected to the sensorial power; the occasional excitement of the muscles of involuntary motion, by which under certain circumstances the sensorial power is also capable of impressing them through the nerves, particularly when under the influence of the passions; the act of causing an evolution of caloric from the blood, by which the due temperature of the animal body is maintained; the act of forming from the blood the various secreted fluids, and of maintaining the other assimilating processes by which the healthy structure of every part of the body is preserved.

The first of these functions is universally acknowledged to be a function of the nervous power, properly so called; but there has been much difference of opinion respecting the way in which it operates. The older physiologists believed that the muscles derive their power from the nervous system. HALLER* was the first who taught that the muscular power belongs to the muscle itself, to which the nervous power bears no relation but that of a stimulus, and endeavoured to support those opinions by experiment. His opponents, however, objected to his inferences, because, although the division of the nerves may prevent the muscle from receiving more nervous power, it does not deprive it of that already bestowed upon it, either existing in the muscular fibres themselves, or dispersed through them in nerves too small to be removed; and this objection appeared to be strengthened by the muscles of involuntary motion,

* Element. Physiolog.

whose function is supported by stimuli peculiar to themselves, being still supplied with nerves, of the use of which HALLER gave no satisfactory account. It appeared to me that the question could only be determined by some experiment capable of directly ascertaining whether the excitability of muscles is maintained by the influence they receive from the nerves, or impaired as by other stimuli. On trial, the latter was found to be the case. Muscles whose nerves had been divided, sustained the action of the same stimulus longer than those whose nerves were entire, and which consequently were exposed to the action both of the nervous power applied by the will of the animal and the artificial stimulus*. The power of the muscle, therefore, is independent of the nervous power, and is affected by it in the same way as by other stimuli.

The experiments by which all the other functions just mentioned, with the exception of the maintenance of animal temperature, have been ascertained to be functions of the nervous power, I have laid before the Society, which has done me the honour to publish them. From these experiments it appeared that the functions in question were always destroyed by depriving their organs of the influence of the nervous system. That the maintenance of animal temperature is a function of the nervous system, properly so called, appears from a variety of facts generally known, the temperature either of a part or of the whole body being lessened by any cause that impairs the action of particular nerves in the former instance, or of the whole nervous system in the latter. The question then is, is the nervous system capable of all these functions after the sensorial power is withdrawn?

At the moment of what we call death, the sensorial functions cease, the animal no longer feels or wills. Whether the nervous functions properly so called still continue, can only be determined by experiment. That the nerves when stimulated are still capable of exciting the muscles of voluntary motion is a fact generally admitted; and that they are still capable of exciting the action of the muscles of involuntary motion, appears from many experiments related in the second paper, which I had the honour to present to the Society, and which was published in the Philosophical Transactions of 1815. That the nervous system is capable of causing the evolution of caloric, which supports animal temperature after the sensorial power is withdrawn, appears

* My Treatise on the Vital Functions, third edition, Exper. 34, 35.

from many experiments related in my treatise on the Vital Functions; and that the nervous power under the same circumstances is still capable of forming the secreted fluids, and supporting the other processes by which the structure of every part is maintained, is shown by very frequently repeated experiments on the newly dead animal related in the same treatise. From these experiments it appears that some secretion of gastric juice takes place after what we call death, and that some derangement of structure in the lungs may be produced by dividing the eighth pair of nerves immediately after death; a proof that the processes on which the structure of the part depends, continue for some time after the sensorial power can no longer influence them.

We may thus trace the existence of the whole of the nervous functions properly so called after the removal of the sensorial power. The former therefore have no immediate dependence on the latter; but in the entire animal we know that the nervous, in many of its functions always, and occasionally in all of them, is subjected to the sensorial, power. These powers therefore bear the same relation to each other that the nervous and muscular powers do, the muscular existing independently of the nervous, but being influenced by it.

It was this independence of the functions properly called nervous on those of the sensorial power, and the analogy which subsists between the former and chemical processes, which suggested that the agent, on which the nervous functions immediately depend, instead of being peculiar to the living animal, may only be an agent employed by those powers which are so, in the same way as any other constituent part which the living animal possesses in common with inanimate nature; and it appeared to me that the accuracy of this suggestion would be placed beyond a doubt if the nervous power could be proved to be capable of its function, after it had been made to pass through any other conductor than the nerves; for it will be admitted that the powers peculiar to the living animal can only operate, and, as far as we see, can only exist in the organs to which they belong. The brain cannot perform the office of a muscle, nor a muscle that of the brain.

If then the nervous power can be made to pass through any substance but that of the nervous system in which it resides, it evidently has an existence independent of the mechanism of that system, and therefore is not peculiar to it. This, after many vain attempts, I succeeded in effecting. It appears from ex-

periments, an account of which the Society did me the honour to publish in 1822, and which have been repeated with the same result by M. Brechet and other physiologists at Paris, that the nervous power is capable of its function after it has been made to pass through other conductors than the nerves.

It would seem, therefore, that however generally the nervous power has been confounded with those powers more strictly called vital, it is only an agent employed by them. This view of the subject seemed to point out the possibility of finding some of those powers which operate in inanimate nature capable of the functions of the nervous power properly so called, if brought to operate under the same circumstances; and on trial it was found, as appears from experiments published in the Philosophical Transactions of 1822 and 1828, and repeated with the same result by Dr. Abel*, M. Brechet† and others, that galvanism may be substituted for the nervous power, not only in the more simple, but in the more complicated functions of that power. It not only appears that galvanism is capable of exciting the muscles and causing an evolution of caloric from arterial blood‡, but of forming the secreted fluids from the blood, and supporting all those functions on which the structure of the body depends. How far do the whole of these facts, whether relating to the nature or functions of the nervous power, go in proving its identity with galvanism?

On reviewing what has been said of the relations of the sensorial, nervous, and muscular powers, the question naturally arises; If both the nervous and muscular powers are thus independent of the sensorial power, and capable of their functions after it is withdrawn, why do the more perfect animals for so short a time survive the loss of the sensorial functions? The cause is, that on the removal of the sensorial power, respiration ceases; because this function partakes of all the three powers, the sensorial, nervous, and muscular.

It has been customary to speak of the muscles of respiration as at least in part muscles of involuntary motion. What is meant by a muscle of voluntary

* The London Medical and Physical Journal for May 1820, vol. xliii. p. 385.

† De l'Influence du Système Nerveux sur la Digestion Stomacale; par MM. Breschet, D.M.P., chef de Travaux Anatomiques de la Faculté de Medecine de Paris, etc.; H. Milne Edwards, D.M.P.; et Vavasseur, D.M.P. (Mémoire lu à la Société Philomatic la 2 Aout, 1823.) Extrait des *Archives Generales de Medecine*, Aout 1823.

‡ My Treatise on the Vital Functions, third edition, Exper. 80, 81, 82, 83, 84, 85, 86.

motion? It is a muscle whose action under all ordinary circumstances we can excite, interrupt, retard, and accelerate at pleasure, but it is not a muscle whose action we can at all times controul. There is no such muscle, because the impression on the sensorium tending to call any particular set of muscles into action may be so powerful, that we are unable to controul it. Who can prevent the action of the muscles of the arm when fire is suddenly applied to the fingers? Neither do we mean by the term muscle of voluntary motion, one which we cannot call into action during sleep. If our posture during sleep becomes uncomfortable, we call the muscles both of the trunk and limbs into action for the purpose of changing it. The uneasiness caused by the continuance of the same posture, sufficiently rouses the sleeper to make him will a change of posture, without rendering him at all more sensible to other impressions of a slighter nature, and his sleep continues.

What muscles then are more under command than those of respiration? We can on all usual occasions interrupt, renew, retard, or accelerate their action at pleasure; and if we cannot interrupt it for as long a time as that of the muscles of a limb, this depends on no peculiarity in the action of these muscles, but on the nature of the office they are called on to perform; and if we excite them in sleep for the removal of an uneasy sensation, and cannot controul them under a sense of suffocation, that is, in a state of greater suffering than we can voluntarily bear, all this is no more than applies to every other muscle of voluntary motion: but from the nature of our constitution we must breathe many times every minute, and we need not turn ourselves more than once in many hours,—a difference depending on circumstances which have nothing to do with the nature of the muscles we employ in either of these acts.

If we find the breathing going on in apoplexy after all voluntary motion of the limbs has ceased, it is because the sensation exists which calls on the patient to inflate his lungs, while there is none which calls for the action of the limbs. In the slighter states of apoplexy if the limbs be much irritated, the muscles which move them will also be called into action; and in the severer states, if the patient breathes, when no irritation of the limbs can excite him to move them, it is that the want of wholesome air in the lungs, after a certain interval, produces a more powerful impression than any other means we can employ. People have voluntarily held the hand in the fire, but no man ever

voluntarily abstained from breathing till the lungs were injured. When at length no irritation, however violent, can impress the sensorium, the breathing ceases and death ensues. The mode of death sufficiently illustrates what is here said. We find the intervals of breathing becoming longer before it ceases. As the insensibility increases, a greater want of fresh air is necessary to excite the patient to inspire, till at length the total privation of fresh air no longer producing any sensation, can no longer excite this effort.

The muscles of respiration then, it would appear, are as perfectly muscles of voluntary motion as those of the limbs, and are never excited but by an act of the sensorium. When there is no feeling to induce us to breathe, the breathing ceases.

That on ordinary occasions we are unconscious of this feeling, in the common acceptation of the term, (that is, that it makes no lasting impression on the mind, for this is necessary to what we mean by consciousness,) unless the attention is particularly directed to it, is no proof that it has not existed. When we direct our attention to the act of breathing, especially if we breathe more slowly than usual, we can distinctly perceive the sensation which induces us to inspire, and that it is a voluntary act which relieves it.

The same observations respecting consciousness apply to all the more trivial habitual acts of the sensorium. In playing on an instrument, we cannot tell which finger last struck the chord; in walking, we cannot tell which leg we last moved;—yet all such acts are strictly acts of volition: when we attend to them we can regulate them as we please, but in proportion as they are habitual we attend to them the less, and therefore least of all to the act of respiration.

To the consciousness of having experienced any feeling, it is evident that its strength, or some other circumstance attending it, must be such as to impress it on the memory. We are every hour performing many acts of volition which are too trivial to be remembered, and consequently at the time we are questioned we have no consciousness of their having existed. The proper feeling excites the act required, but the feeling is too habitual to command the attention.

It may be difficult for a person not accustomed to reflect on such subjects, to believe that every time his leg is moved in walking, he performs a distinct act of volition; but he will be convinced of this if he observes the motions of

those whose power of volition is impaired by disease. He will find the patient hesitate which leg to move at every step, and at length his attempts to move the limbs produce a confused and irregular action incapable of carrying him forward.

The act of expanding the chest is an act of volition, it is an act in ordinary breathing rendered extremely easy by the gentleness of the motion required, and the continual habit which renders it familiar, and is excited by a sensation proportionably slight, but which is as essential to it as stronger sensations are to more powerful acts of volition. Thus it is that on the removal of the sensorial power respiration ceases. It may be here said perhaps, that we have no instance of a muscle of voluntary motion continuing to act at short intervals during life; but besides that this is begging the question, it is to be recollected that the action of the muscles in ordinary respiration is very slight, and performed at considerable intervals, for it is only during inspiration that the muscles act. They are quiescent during expiration, which in our usual breathing is performed by the elasticity of the cartilages and the weight of the parts concerned. There is perhaps no muscle of the body which could not without fatigue maintain a similar action were there a cause capable of exciting it. In certain diseases we find both more powerful and more frequent actions of the muscles of volition continued for years during the whole of our waking hours without any complaint of fatigue.

When the change in the blood, effected by respiration, no longer takes place, most of the pulmonary vessels lose their proper stimulus, red blood; and feel more directly perhaps the debilitating influence of black blood; their functions therefore begin to fail. In proportion as this happens, the blood accumulates in the lungs. The right side of the heart consequently experiences an increased difficulty in emptying itself, and the due supply of blood to the left side fails. By the operation of these causes both sides of the heart, particularly in warm-blooded animals, soon lose their power after respiration ceases. The arteries under such circumstances, it is evident, cannot long supply fluids proper for the purposes of assimilation. The nervous and muscular solids therefore deviate from the state necessary for the functions of life, which at length cease in every part.

The foregoing appears to be the order in which the functions always, with

the exception of their instantaneous destruction as above mentioned, cease in death; whether it be occasioned by injury of the sanguiferous or nervous system, or both.

Such then appears to be the nature of respiration. The first act is the impression made on the sensorium, the sensation excited by the want of fresh air in the lungs. We are enabled to supply it, and thus remove the uneasiness, by exciting certain muscles subjected to the will. Through nerves which are fitted for this purpose, we apply a stimulus to certain muscles which perform the act required. Thus respiration is the combined act of the sensorial nervous and muscular powers. It is as effectually destroyed by a failure of the sensation which makes us will to inspire, as by that of the nervous or muscular power by which the will effects its object. With this view of the subject before us, and I can see no other which the facts admit of, it will be proper to examine the nature of respiration more in detail.

I have already had occasion to observe, that the effort made in ordinary breathing is very slight. It is chiefly performed by the diaphragm, by the contraction of which the cavity of the chest being slightly enlarged perpendicularly, the pressure of the atmosphere readily causes the air cells to be distended with air; but if any obstacle occurs tending to prevent the passage of the air to the cells, a greater effort is required, and other muscles are called into action. It seems almost unnecessary to observe, that the sensation which induces us to make this greater effort, must, as the object is still the same, operate in the same way. The more powerful sensation indeed, and the trouble the effort gives us, by calling our attention to it, enables us at once to perceive that it is an effort of the same kind with any other voluntary effort by which we endeavour to relieve ourselves from a painful feeling, and, like any other powerful voluntary effort long continued, produces the feelings of fatigue. Would any privation of air induce the struggle that we see in severe dyspnœa, if no sensation were excited by it? This sensation is excited in the sensorium through the nerves of the lungs, and all that follows is evidently the result of it.

The effort consists in two things, drawing the air into the chest with greater force, that is, expanding the chest more forcibly that the air may enter it with a greater degree of atmospheric pressure, and thus any obstacle to its entrance be overcome; and doing all we can to enlarge the passage by which the air enters.

The action of the muscles by which these objects are effected has been ascribed to a particular sympathy supposed to exist between certain nerves. But if the eighth pair of nerves which supplies the lungs originate near the nerves of the diaphragm, and certain muscles of the face, by which the nostrils are expanded, this cannot be said of the nerves of many other muscles equally called into action in severe dyspnœa, the muscles of the loins, &c.; and if we could by what is called sympathy of nerves explain the phenomena in question, it is not to be overlooked that the same sympathy must exist with respect to the abdominal as thoracic viscera, for the same nerves supply both.

We must therefore look for another principle to account for the relation which subsists between such acts and peculiar states of the lungs. The principle is at hand. The sensation which induces us to inspire forms a necessary link in the chain of causes; for every contraction excited in the muscles is evidently calculated to relieve this sensation in one of the two ways just pointed out. It either tends to expand the chest, or enlarge the passage of the air. It is impossible in such a case to overlook the act of the sensorium, which is sufficient to account for the phenomena without any particular sympathy of nerves, which on the other hand, I have just had occasion to point out, is insufficient for this purpose.

The muscles employed in extreme dyspnœa are not confined to a particular set. They are the whole muscles of the trunk, and sometimes many of the limbs also, muscles which have nothing in common, except that they are all muscles of voluntary motion, and bear the same relation to the nervous and sensorial systems which all other muscles of voluntary motion do. Actions of the muscles of the face indeed are equally associated with sensations referred to the abdomen and the limbs, and arising from causes operating in them. Who can have a placid countenance while in agony from the operation of any cause to whatever part applied?

It appears from a great variety of experiments to which I have referred, that organs supplied with ganglionic nerves are subjected to the influence not of any one, but of every part of the brain and spinal marrow. No inference therefore can be drawn respecting the sympathies of any ganglionic nerve, as the term is here used, that is a nerve that either enters or proceeds from ganglions, according to the sense in which I use the term, from any particular distribution of nerves, or from the part where any particular nerve which con-

tributes to the power of the ganglionic system originates. Vital organs are equally connected with every part of the brain and spinal marrow; and if we must not look for those partial sympathies with respect to their other functions, there is still less room, it is evident, to look for them in those functions where the sensorial power is concerned.

The sensorium evidently residing and operating at the source of nervous power, there receives the various impressions conveyed by the nerves, and there influences those nerves which convey its dictates.

I shall beg leave to conclude this paper with a short recapitulation of the principal points which appear to be ascertained by the experiments referred to in it.

The nerves are divided into two classes, whose functions essentially differ; those proceeding directly from the brain and spinal marrow, which, in the one direction, convey the influence of the parts of those organs from which they have their origin, and are the sole means of exciting the muscles of voluntary motion; and in the other, impressions which influence the sensorium: and the ganglionic nerves, which, while they also convey impressions to the sensorium and occasionally excite the muscles of involuntary motion, usually excited by stimuli peculiar to themselves, have for their principal function one of greater importance, and which requires the combined influence of the whole brain and spinal marrow, that of supporting the various processes of secretion and assimilation, and are consequently in the strictest sense a vital organ.

Although the nervous power therefore stands only in the relation of a stimulus to the muscular fibre, whether of voluntary or involuntary motion, in no degree contributing to its power, which depends on its own mechanism; it is essential to the existence of the secreting and assimilating powers, which are immediately destroyed by withdrawing its influence.

Such is the relation which the nervous system bears to what may be termed the circumference of the animal body, in contradistinction to the sensorium, which may be justly regarded as its centre, to which that system bears a relation of equal importance; for it may be regarded as the means of connecting the organs of the sensorium with all other parts. In its power this system is independent of the sensorium, for we have seen it capable of all its functions after the sensorial power is withdrawn; but in all of them it is in-

fluenced by it, constantly in some, occasionally in others. It therefore bears the same relation to the sensorial organs which the muscles bear to it. As the muscular is independent of the nervous power, so is the nervous of the sensorial power. As the nervous, influence all the muscular, functions, those of the muscles of voluntary motion constantly, those of the muscles of involuntary motion occasionally; so the sensorial, influence all the nervous, functions, those of the cerebral and spinal nerves constantly, those of the ganglionic nerves occasionally. Thus all the functions of the nervous and muscular systems, by which we are connected with the world that surrounds us, are constantly subjected to the sensorial power; while the functions on which our life depends, with the exception of respiration, are only occasionally so, and under circumstances in which the will has no controul. With this exception the latter are all functions of the nervous and muscular powers alone. To respiration the sensorial power also is necessary, and therefore the nervous and muscular powers never long survive the loss of the sensorial power.

The nervous power which connects all the other powers of the animal body, effects so many changes in it, and has so large a share in connecting it with the world around it, cannot strictly speaking be regarded as one of the vital powers of that body, but as an agent employed by those powers; because it has been proved by direct experiment that it is capable of existing independently of the mechanism of the part in which it resides, and therefore is not peculiar to that mechanism; and by the same means, that all its functions may be performed by galvanism, made to operate in the same circumstances in which the nervous power operates.

The experiments referred to in the foregoing paper suggested the use of galvanism in those diseases which arise either from a partial or general failure of the nervous power; and the success which has attended its employment has afforded another proof of its capability of the functions of that power. The diseases in which it has been chiefly employed are habitual asthma, the various forms of indigestion, affections of the spinal marrow and general nervous debility. An account of its effects in the first of these diseases was laid before the Society, and published in the Philosophical Transactions of 1817. An account of its effects in the others is published in the third edition of my treatise on the Vital Functions.

XXII. *On the respiration of birds.* By William Allen *and* William Hasledine Pepys, *Esqrs. Fellows of the Royal Society.*

Read April 30, 1829.

OUR communications to the Royal Society, as printed in the Phil. Trans. for 1808 and 1809, detailed the effects produced when the human subject or a guinea-pig respired, either atmospheric air alone, or pure oxygen, or a mixture of hydrogen and oxygen. We thought it would render the subject more complete if we extended our inquiries to the respiration of birds, and accordingly made several experiments with pigeons in the same apparatus that we employed for the guinea-pig. The apparatus is engraved and described in the Phil. Trans. for 1809, page 429.

First experiment with atmospheric air.

A pigeon was placed in the intermediate glass vessel, in 62 inches of air on the mahogany stand over quicksilver, between the two gasometers communicating with the vessel in which the pigeon was confined. One of the gasometers was empty, but connected by tubes and stop cocks with the quicksilver bath, and also with the intermediate vessel; the other contained the air for the supply of the pigeon. The barom. being 30.130, the therm. 54°, during 69 minutes, at intervals of four or five minutes, 35 cubic inches at a time of common air were slowly passed through the vessel in which the bird was confined; the other gasometer of course received what was pushed off, the quantity was noticed by its register, and a portion was received by a bottle in the quicksilver bath for examination; in this way 525 cubic inches of common air were supplied, to which the 62 cubic inches in the intermediate glass vessel being added, made a total of 587 cubic inches in which the pigeon had respired for 69 minutes. The registers of both gasometers agreed throughout to a very trifle, which confirmed our former observations, that there is no change in the volume of atmospheric air when respired under natural circumstances.

At the end of the first twelve minutes we noticed a good deal of dew upon the glass opposite to the head of the pigeon; at first we gave a fresh supply of air every five minutes, but at the end of 31 minutes the bird appearing a little uneasy, we supplied him every four minutes; and at the close of the experiment, which lasted 69 minutes, he did not appear the worse for it. We thought that the register indicated a slight diminution of volume during the time that the bird was uneasy; the air respired was examined as usual, with lime water in the eudiometer with an elastic bottle, for carbonic acid, and with green sulphate of iron saturated with nitrous gas, for oxygen.

State of the air before the experiment: Atmospheric air 587 cubic inches, consisting of 123 oxygen, 464 azote.

State of the air after the experiment; 587 cubic inches, consisting of 35.80 carb. acid, 87.52 oxyg., 462.67 azote. $35.80 \div 69 = .52$ cubic inches per minute. Thus it appears that the bird produced about half a cubic inch of carbonic acid per minute; and as the volume of oxygen consumed is always equal to the volume of carbonic acid produced, the 35.80 being added to the oxygen found after the experiment, or 87.52 cubic inches, very nearly corresponds with the 123 contained in the common air before the experiment. Now as 100 cubic inches of carbonic acid contains 12.82 grains of carbon, the 35.80 cubic inches produced by the bird in 69 minutes must contain 4.59 grains of carbon, or nearly at the rate of 96 grains in 24 hours.

The azote before the experiment was 464 cubic inches; after the experiment 462.67: the difference is only 1.33, or little more than a cubic inch, which loss might have happened during the time the bird was uneasy; and we may fairly conclude that when birds respire atmospheric air, the only change in the air is, the conversion of a part of its oxygen into a corresponding portion of carbonic acid gas.

First experiment with oxygen gas.

The oxygen was made from chlorate of potash, and being examined as usual with the eudiometer, it was found to contain 2 per cent of azote.

The barom. being 29.5, the therm. 51°, we placed the pigeon in the intermediate glass vessel in the same situation as in the former experiment.

The capacity of the glass vessel was	94 cubic inches.
Connecting tubes, &c.	5
	99
Bulk of bird	28
Atmospheric air	71

Thus it appears that 71 cubic inches of atmospheric air were contained in the glass vessel with the pigeon, and during 22 minutes 75 cubic inches of oxygen gas were passed through this vessel at intervals of about 7 minutes, the quantity pushed off being noticed by the register of the second gasometer.

In about ten minutes the bird began to pant and became uneasy; during the next 21 minutes 72 cubic inches more of oxygen gas were passed: the bird continued to pant, and the soft parts about his beak became of a bright red.

In the next 23 minutes 89 cubic inches more of oxygen were passed, and the bird was left 6 minutes longer in the 71 cubic inches contained with him in the intermediate glass vessel, making the whole time one hour and twelve minutes; the bird on being released appeared very well, and did not seem at all injured by the experiment. The gas being examined by the eudiometer as before, the following results were obtained:

Vol. of gas before the expt. cub. inch.		Azote.	Oxygen.	Time.	Vol. of gas after expt.		Azote.	Oxygen.	Carbonic Acid.
71	atmos. air consisting of	56.69	14.31						
75	oxyg. with 2 per cent azote	1.50	73.50	22′	75	consisting of	48.22	22.96	3.82
72	oxyg. ditto	1.44	70.56	21	71.99		23.75	43.79	4.45
89	oxyg. ditto	1.78	87.22	23	89		10.90	70.90	7.20
			Residuum	6	71		7.24	57.96	5.80
307		61.41	245.59	72	306.99		90.11	195.61	21.27

Thus it appears that the volume of the gas was almost unaltered, but that there had been a great disturbance in the proportions of the azote.

Azote in the 307 cubic inches of gas:

Before the experiment	61.41
Found after the experiment	90.11
Increase of azote	28.70

Oxygen in the 307 cubic inches of gas:

Before the experiment		245.59
Found after experiment	195.61	
In carbonic acid gas	21.27	
		216.88
Loss of oxygen		28.71

This agrees with the facts stated in our former paper in the experiment with the guinea-pig; but there is a striking difference in the case of the carbonic acid; so far from there being an increase of it when the pigeon breathed oxygen, there was a considerable diminution. $21.27 \div 72' = .29$ of a cubic inch per minute, which is little more than a quarter of a cubic inch per minute; but he produced more than half a cubic inch in the same time when he breathed common air; and with regard to the increase of the azote, we may remark that at the end of the first period of 22 minutes, rather less than ten cubic inches of azote were left, for

$$\left.\begin{array}{l} 56.69 + 1.50 = 58.19 \text{ azote before} \\ \text{and} \qquad\quad 48.22 \text{ azote after} \end{array}\right\} = 9.97 \text{ of the original azote left.}$$

In the following period of 21 minutes none of the original azote was left, but there was an increase of about 13 cubic inches. In the next period of 23 minutes only 1.44 of azote was admitted, but 10.90 were found; thus $10.90 - 1.44 = 9.46$ of azote evolved in that time; it seems therefore plain that oxygen must have been absolved by the blood in the lungs, and a corresponding volume of azote given out.

Second experiment with oxygen gas.

The oxygen gas was made as before from chlorate of potash, and contained in this instance only one per cent of azote.

Barom. 30.50, therm. 45°. The quantity of atmospheric air with the pigeon in the intermediate vessel was 69 cubic inches; during the first 20 minutes 62 cubic inches of oxygen were supplied. In about a quarter of an hour he began to open his beak, pant, shake his head and appear uneasy, frequently drawing the film over his eyes, and often putting out his tongue, the respiration becoming quicker. When he had been in about 40 minutes his beak reddened

very much, and he opened it every time he breathed; there was more moisture on the glass opposite to his head, than in the experiment with common air. Towards the conclusion of the experiment, which lasted one hour and ten minutes, the film was pretty constantly over his eyes, but on being taken out he was quickly as well as usual.

The following was the state of the gas before and after the experiment:

Vol. of gas before the expt. cub. inch.		Azote.	Oxygen.	Time.	Vol of gas after expt.		Azote.	Oxygen.	Carbonic Acid.
69	atmos. air =	54.51	14.49						
62	oxyg. 1 per cent =	.62	61.38	20′	76.5	=	42.33	29.90	4.27
86	oxyg. =	.86	85.14	21	75	=	19.80	48.73	6.47
84	oxyg. =	.84	83.16	29	73.5	=	8.56	58.43	6.51
					7	=	.71	5.60	.69
			Residuum		69	=	7.	55.20	6.80
301		56.83	244.17	70	301.0		78.40	197.86	24.74

Here again the volume of gas appears to be unaltered: the production of carbonic acid gas is but little more than in the last experiment,

$$24.74 \div 72 = .35 \text{ cubic inch per minute;}$$

but the proportions of azote are altered as in the former experiment: thus

Azote in the 301 cubic inches of gas.

Before the experiment	56.83
Found after the experiment	78.40
Increase of azote	21.57

Oxygen in the 301 cubic inches of gas.

Before the experiment		244.17
Found after the experiment	197.86	
In carbonic acid gas	24.74	
		222.60
Loss of oxygen		21.57

It is interesting to notice here also the state of the azote at different periods of the experiment.

In the first period of 20 minutes the azote before respiration was

54.51 + 62 = 55.13

Azote found 42.33

Azote left 12.80

Second period of 21 minutes.

12.80 + 86 = 13.66

Azote found 19.80

Increase of azote 6.14

Third period of 29 minutes.

Azote in 84 cubic inches of oxygen ... .84

Found ..... 8.56 + .71 + 7 = 16.27

Increase of azote 15.43

15.43 + 6.14 = 21.57 total increase of azote. Here the loss of oxygen exactly equals the increase of azote.

Mixture of hydrogen and oxygen.

We next tried the effects of an artificial atmosphere, substituting hydrogen for azote; the oxygen made as before contained 3 per cent of azote, and detonating 10 parts of it with 20 of hydrogen in VOLTA's eudiometer, the whole volume of gas disappeared except about one part which was azote, and this confirmed the result with the other eudiometer and green sulphate of iron saturated with nitrous gas.

There were but 64 cubic inches of common air this time in the intermediate vessel containing the pigeon at the beginning of the experiment, and 187 cubic inches of a mixture of hydrogen and oxygen, in which the oxygen was in about the same proportion to the hydrogen as it is to the azote in the common air, were gradually passed through the intermediate vessel during 26 minutes.

Duration of the expt.

at 55′ after 2 P.M. the pigeon was put into the vessel with common air; moisture almost immediately began to condense on the inside of the glass.

Duration of the expt.

6′ at 1′ after 3 P.M. made 35 cubic inches of the mixture pass slowly through the vessel.

10 at 5 after 3 P.M. passed 35 cubic inches more; the bird became uneasy.

15 at 10 after 3 P.M. passed 35 cubic inches more.

18 at 13 after 3 P.M. the bird now panted very much, but was relieved as soon as more of the mixture was admitted, passed 35 cubic inches.

22 at 17 after 3 P.M. passed 35 cubic inches more.

23 at 18 after 3 P.M. passed 12 inches; being the last the bird struggled.

26 at 21 after 3 P.M. took it out, but it did not seem to be in the least injured.

The following was the state of the gas before and after the experiment:

Vol. of gas before the expt. cub. inch.		Azote.	Oxygen.	Hydrog.	Time.	Vol. of gas after expt.		Azote.	Oxygen.	Carbonic Acid.	Hydrog.
64.00	atmos. air =	50.56	13.44								
39.27	oxygen 3 per cent =	1.18	38.09								
147.73	hydrogen =			147.73	26′	250.99	=	86.97	34.15	17.62	112.25
251.00		51.74	51.53								

Azote in the 251 cubic inches of gas.

Before the experiment	51.74
Found after the experiment	86.97
Increase of azote	35.23

Oxygen in the 251 cubic inches of gas.

Before the experiment		51.53
Found after the experiment	34.15	
In carbonic acid	17.62	
		51.77

The volume nearly the same after as before the experiment.

Hydrogen in the 251 cubic inches of gas.

Before the experiment	147.73
Found after the experiment	112.25
Loss of hydrogen	35.48

17.62 ÷ 26′ = .68 cubic inches per minute.

The production of carbonic acid in this case was somewhat greater than in atmospheric air; but the remarkable feature of this experiment is, that, except in the formation of carbonic acid, the oxygen remains nearly unchanged, while the whole loss falls upon the hydrogen; so that the blood appears to have absorbed a quantity of hydrogen, and given out a proportionate quantity of azote, the total volume of gas before and after the experiment remaining nearly the same.

The present experiments tend to strengthen and confirm our former conclusions, and prove that when atmospheric air alone is respired in a natural way, the proportion of azote is not altered, and that there is only a change of a certain portion of oxygen for an equal portion of carbonic acid gas; that when a larger proportion of oxygen than is contained in atmospheric air is respired, a quantity of oxygen is absorbed by the blood, and an equal quantity of azote gas evolved.

That when a mixture of hydrogen, oxygen, and azote are respired, the oxygen being in the same proportion as in atmospheric air, there is no loss of oxygen, but a quantity of hydrogen disappears and is replaced by the same quantity of azote.

The circulation of the blood is quicker in birds than in other animals; and if we may judge from the effects produced upon the pigeon, they are more sensible to the stimulus of oxygen.

XXIII. *On the spontaneous purification of Thames water.* By JOHN BOSTOCK, *M.D. F.R.S. &c.*

Read April 30, 1829.

IN the Report respecting the analysis of the water of the Thames, which I presented, in April 1828, to the Commissioners appointed by His Majesty to inquire into the supply of water in the Metropolis, I have stated that when the experiments were nearly brought to a close, a quantity of water was sent to me, purporting to have been "taken in the river, in the current of, and immediately at the mouth of the King's Scholars' Pond sewer." I described it as "in a state of extreme impurity, opaque with filth, and exhaling a highly fœtid odour." When it had been about a week in my possession, a considerable quantity of black water subsided from it, but the fluid was still dark-coloured and opaque, and nearly as offensive as at first, while the odour and colour were only in part removed by being passed through a layer of sand and charcoal, six inches in thickness.

The water remained for some time in my laboratory without being attended to; when after an interval of some weeks, I observed that a great change had taken place in its appearance. It was become much clearer, whilst nearly the whole of the sediment had risen to the surface, where it formed a pretty regular stratum of about half an inch in thickness; the odour, however, still continued extremely offensive, perhaps even more so than at first. From this time the process of depuration, which had thus spontaneously commenced, was continued for about eight weeks, when the water became perfectly transparent, without any unpleasant odour, although still retaining somewhat of its original dingy colour.

After the formation of the scum mentioned above, the next change that I observed was its separation into large masses or flakes; to these, as well as to the scum itself, a number of minute air bubbles were attached, to which, no

doubt, they owed their buoyancy; after some time the masses again subsided, leaving the fluid almost totally free from any visible extraneous matter. The quantity of gas discharged was inconsiderable, so that it was difficult to obtain any of it for examination. It seemed to be principally composed of carbonic acid, containing a little sulphuretted, and perhaps carburetted, hydrogen gas.

When the process of depuration appeared to be complete, the water was filtered through paper, and was then subjected to the same mode of analysis which was employed on the former occasion*. It was now perfectly transparent, and without taste or odour, but still retaining a slight brown tinge. It sparkled when agitated or poured from one vessel to another, and by boiling a quantity of gas was disengaged from it: at the same time a thin film of carbonate of lime formed on the surface, which gradually subsided: 10,000 grains left by evaporation a saline crust, of a light brown colour, which, after being thoroughly dried, weighed 7.6 grains. By the appropriate tests, the water was found to contain lime, sulphuric acid, muriatic acid, and magnesia. There was a trace of alumine and an indication of potash; but no ammonia, sulphur, or iron could be detected. The lime, the magnesia, and the sulphuric and muriatic acids were all of them obviously in much greater quantity than in the specimens of the Thames water previously examined. If we suppose the sulphuric acid to be combined with a part of the lime, and the remainder of the lime to be in the state of carbonate, and that a part of the muriatic acid is combined with the magnesia and the remainder with soda, as was conceived to be the case in the Thames water generally, the respective quantities of these salts in 10,000 grains will be as follows:

	grs.	grs.	
Carbonate of lime	4.20	1.55	Salts contained in the Lambeth water, which was considered as the most impure of the specimens formerly examined.
Sulphate of ditto	.66	.12	
Muriate of soda Muriate of magnesia	2.74	.23	
	7.60	1.90	

The result of this analysis shows, that although the water has, by this depurating process, freed itself from the great quantity of organic matter which it

* Report, p. 80—81.

contained, and acquired a state of apparent purity, which might render it sufficiently proper for many purposes, yet that the quantity of saline matter is increased as much as four-fold. The greatest proportionate increase is in the muriates, which are very nearly twelve times more in the purified water than in the Thames water in its ordinary state. The carbonate of lime is between two and three times as abundant as before, and the sulphate of lime between five and six times. I may remark, that this water, when examined in its foul state, gave very obvious indications of both sulphur and ammonia, neither of which could be detected after depuration.

This depurating process may be denominated a species of fermentation; *i. e.* an operation, where a substance, without any addition, undergoes a change in the arrangement of its component parts, and a new compound or compounds are produced. The newly formed compounds were, in this case, entirely gaseous, and, except a part of the carbonic acid, were discharged. The saline bodies, being not affected by this process, remained in solution, leaving the fluid free indeed from what are considered as impurities, yet so much loaded with earthy and neutral salts, as to be converted from a soft into a hard water*. The source of the saline bodies may be supposed to be the organic substances, principally of an animal origin, which are so copiously deposited in the Thames; of these the most abundant are the excrementitious matters, as well as the parts of various undecomposed animal bodies. The different species of the softer and more soluble animal compounds act as the ferment, and are themselves destroyed, while the salts which were attached to them are left behind. It may be conceived therefore, that the more foul is the water, the more complete will be the subsequent process of depuration; and we have hence an explanation of the popular opinion, that the Thames water is peculiarly valuable for sea stores, its extreme impurity inducing the fermentative process, and thus removing from it all those substances which can cause it to undergo any further alteration.

The brown colour which the water exhibited after its depuration appeared to depend on the solution of a minute quantity of what is generally termed extractive matter, and which is observed in water that contains decayed vege-

* The terms hard and soft, as applied to water, are obviously relative; but water which contains as much as 5 grains in the pint of saline matter, is generally regarded as too hard for many œconomical and manufacturing processes. The water in question contained 4.36 grains per pint.

table substances; it is almost always present in the beginning of winter in the water of ponds, or of slow streams that have received the falling leaves. After the heavy rains that occurred in December 1827, the New River water, with which my cistern is supplied, was observed to be very turbid and dark-coloured. By remaining some hours at rest, a quantity of earthy matter subsided, and left the water nearly transparent, but the dark colour still continued*.

I found that this colouring matter was not removed by boiling, nor by filtration through sand and charcoal, but that alum and certain metallic salts, especially when heated with it, threw down a precipitate, and left the water without colour. Of the metallic salts the most effectual appeared to be the sulphate of iron; a drop of the solution of this salt, boiled with 500 times its bulk of the water, threw down a flocculent, orange-coloured precipitate, and left the water perfectly colourless. I obtained the same results, only much less in degree, when these re-agents were added to the Thames water after its depuration.

The sediment which was removed from the water by filtration, as mentioned above, appeared to be a heterogeneous mass of various substances, about $\frac{9}{10}$ths of which was siliceous sand; it also contained a black matter, which gave the whole a dark gray colour, and which was removed by a red heat, a number of fine fibres that looked like animal down, and some large fibres probably of vegetable origin; there were also bits of wood, fragments of coal, and small shining particles of a metallic nature, which seemed to be sulphuret of iron. The mass indeed consisted of all those substances which were casually introduced into the Thames, and which had not been decomposed by the fermentative process. They must of course differ, both in quantity and in quality, in every different portion of the water, so to render it unnecessary to attempt a more minute examination of them; in the present instance, the sediment, when completely dried at a temperature of 200°, was in the proportion of about 9 grains in 10,000 grains of the water.

* It is not easy to institute any exact comparative scale of the shades of brown. An infusion formed by digesting, for 10 days, powdered galls in twenty times their weight of water, and afterwards diluting the infusion with an equal bulk of water, will exhibit a colour nearly similar to that of the New River water in the state in which I examined it.

XXIV. *On the composition of chloride of barium.* *By Dr.* EDWARD TURNER, *Professor of Chemistry in the University of London. Communicated by Dr.* DIONYSIUS LARDNER, *Fellow of the Royal Society.*

[illegible]ad May 14, 1829.

IN taking a review of the present state of chemistry;—of the numerous compounds that have been discovered within a very limited period, and of which many have as yet been but partially or imperfectly examined;—of the results, often discordant, which analysts have obtained;—and of the opposite theoretic views which are prevalent,—it is difficult to avoid suspecting the propriety of opinions that have been thought to rest on the sure basis of correct observation, or doubting the accuracy of analyses conducted by chemists of the highest reputation. The era of brilliant discovery in chemistry appears to have terminated for the present. The time is arrived for reviewing our stock of information, and submitting the principal facts and fundamental doctrines of the science to the severest scrutiny. The activity of chemists should now, I conceive, be especially employed, not so much in searching for new compounds or new elements, as in examining those already discovered; in ascertaining with the greatest possible care the exact ratio in which the elements of compounds are united; in correcting the erroneous statements to which inaccurate observation has given rise; and exposing the fallacy of opinions which partial experience or false facts have produced. Considerable as is the labour and difficulty of such researches, they will eventually prove of great importance to chemical science by supplying correct materials for reasoning; and will sometimes, even in the most familiar parts of analytical chemistry, lead to the detection of errors that had escaped notice, and which vitiate many analyses previously regarded without suspicion. An instance of this kind I shall have occasion to notice in the present communication.

The foregoing reflections have been more immediately elicited by circum-

stances connected with Dr. THOMSON's "First Principles of Chemistry." The celebrated author of that work has attempted to ascertain the equivalents of all elementary substances; and as the result of his labours, has inferred the truth of an ingenious conjecture, suggested some years ago by Dr. PROUT, that the weights of the atoms of bodies are simple multiples of the atomic weight of hydrogen. (Annals of Philosophy, vol. vi. p. 321.) This hypothesis is of so much importance if true, and may give rise to so much error if false, that its accuracy cannot too soon be put to the test of a minute experimental inquiry. The only chemists who to my knowledge have objected on experimental grounds to Dr. THOMSON's support of this hypothesis, are Dr. URE and BERZELIUS; but unfortunately both these gentlemen have written on the subject with such acrimony, and assumed a tone so unusual in scientific controversy, as in a great degree to have destroyed that confidence which their well-founded reputation for sagacity and skill would otherwise inspire. The uncertainty in which this question is still involved, has induced me to investigate it; and the essay which the Royal Society do me the honour to hear this evening, may be viewed as the commencement of a series of essays designed for the elucidation of the same subject. As I shall have occasion on individual points to differ repeatedly from Dr. THOMSON, I embrace this opportunity to declare, that in considering his statements with the freedom required for eliciting truth, I bear towards him no other personal feelings than those of kindness for civility received at his hands, and of respect for a man who has devoted his life zealously and successfully to the promotion of science.

The object of the present essay is to determine the composition of chloride of barium. The frequent employment of this compound in chemical experiments renders an exact knowledge of its constitution peculiarly important; and it has been used so extensively by Dr. THOMSON as a medium of analysis, that an examination of it will afford an excellent criterion of the accuracy of his researches. Dr. THOMSON has employed chloride of barium in ascertaining the equivalent of sulphuric acid, and of not less than thirteen metals and their protoxides; so that if his examination of this substance is inexact, the error will probably affect a large portion of his treatise. Dr. THOMSON has been led by his observations to adopt 36 as the equivalent of chlorine, 70 as that of barium, and 78 as that of baryta. The equivalent of chloride

of barium is therefore 106; and on mixing this quantity of the chloride with 88 parts of sulphate of potash, each being previously dissolved in separate portions of distilled water, he finds that the clear liquid left after the insoluble sulphate of baryta has completely subsided, is not rendered turbid either by muriate of baryta or sulphate of soda. It is hence inferred, that by double decomposition the whole of the baryta has united with all the sulphuric acid, and that all the potash and muriatic acid are contained in solution in the form of muriate of potash. The resulting sulphate of baryta, after being collected and heated to redness, weighed exactly 118 parts; while the muriate of potash, when collected and duly heated, yielded 76 parts of chloride of potassium. It follows from this experiment that 40 is the equivalent of sulphuric acid, and 48 of potash; and on mixing with one equivalent of chloride of barium such a quantity of any soluble sulphate as should produce a similar interchange of elements, the constitution of that salt would be exactly determined.

This leading experiment, from which Dr. Thomson deduces the composition of chloride of barium as well as the atomic weight of baryta, is maintained by Berzelius to be inexact. He prepared chloride of barium and sulphate of potash with the greatest possible care; and on mixing them in the proportion mentioned by Dr. Thomson, he found that a considerable quantity of the former, about 2.25 per cent of the amount employed, remained free in the residual liquid. (Lehrbuch der Chemie, vol. iii. p. 106.) In an answer to this objection, published in the Philosophical Magazine and Annals of Philosophy for last March, Dr. Thomson has maintained the accuracy of his original experiment, stating that it had recently been repeated by six of his practical pupils, and in no case did the residual liquid contain a trace either of sulphuric acid or baryta. I regret that my observations have forced me to a conclusion precisely opposite. I have made the experiment in question repeatedly, with the greatest care, and with perfectly pure materials, and in every instance the result coincided with that obtained by Berzelius. The sulphate of potash which I used was prepared by repeated crystallization from the crystals of that salt as sold by the druggists, and was so pure that I could not detect in it a trace of foreign matter. The chloride of barium was formed by the action of pure muriatic acid on native carbonate of baryta. The resulting solution was

rendered alkaline with pure baryta, in order to precipitate any oxide of iron or manganese which might be present; and the crystals subsequently obtained by evaporation were reduced to powder, boiled in successive portions of alcohol, and fused. The fused chloride was redissolved in distilled water, and again obtained in crystals. This salt dissolved without residue or turbidity in water, and the solution was not affected by pure ammonia; it was not discoloured by sulphuretted hydrogen, hydrosulphuret of ammonia, or chloride of lime; when precipitated by an excess of sulphate of potash, the soluble parts were not rendered turbid by an alkaline carbonate or oxalate of potash; and when thrown down by pure sulphuric acid, evaporated, and ignited, the dry mass did not yield a trace of any soluble sulphate to water. Both compounds were heated to redness before being employed; and the chloride of barium, which if perfectly anhydrous, attracts moisture freely from the atmosphere, was always placed while hot in a weighed bottle secured by a tight cork, and its weight ascertained when cold. This precaution is not necessary with sulphate of potash.

I have thought it right to enter into these details, not only that chemists may judge of the accuracy of my experiments by the care with which they were conducted, but because the error committed by Dr. Thomson appears referable to the neglect of some of these precautions. This opinion seems the more probable, since Dr. Thomson is uncertain whether in his original experiments he did not employ the muriate of baryta of commerce, and if so he doubtless must have operated with an impure substance. But independently of any inaccuracy arising from this source, I shall now endeavour to prove that his method involves an error which precludes an exact result even with the purest materials. When solutions of muriate of baryta and sulphate of potash are mixed together, a small portion of the latter invariably escapes decomposition, and falls tenaciously adhering to the sulphate of baryta. I was led to this fact by observing, that when a known quantity of chloride of barium is precipitated by sulphate of potash, the resulting sulphate of baryta always weighed more than when the precipitation was made with pure sulphuric acid. The appearance of the salts after exposure to a red heat, was likewise different; the impure sulphate being harder, more brittle, and less opaque than the pure sulphate. The former reduced to powder and boiled with water, yielded a so-

lution which precipitated barytic salts freely, and afforded certain evidence of the presence of potash with muriate of platinum.

The presence of sulphate of potash was at first naturally ascribed to imperfect edulcoration; but as it was still found, even after the precipitate had been washed with unusual care, I was led to examine the subject minutely. A solution of sulphate of potash was mixed with a large excess of muriate of baryta; the insoluble sulphate was edulcorated until the washings ceased to contain a trace of baryta, and was then collected on a filter, and ignited. On boiling it in powder with water, sulphate of potash was dissolved. The experiment was varied by mixing the solutions at a boiling temperature, and continuing the ebullition for some minutes; but the result was the same as before. On edulcorating the precipitate with boiling water, sulphate of potash begins to make its appearance in the washings as soon as the excess of muriate of baryta has been removed; but neither by this means, nor by boiling the recent precipitate for hours in successive portions of distilled water, have I succeeded in removing all the sulphate of potash. The adhesion of this salt ensues even in a dilute solution; and it is not prevented by the presence of other salts, such as nitre, and nitrate or muriate of ammonia, nor by free muriatic acid. The quantity of adhering sulphate of potash is variable, depending apparently as well on the relative quantity of the two salts, and the strength of the solution, as on the manner and extent of edulcoration. I have known it to increase the weight of the sulphate of baryta by one per cent.

The foregoing observations, unless I am much deceived, will fully justify the statement, that Dr. Thomson's method of analyzing chloride of barium is radically defective. For if chloride of barium and sulphate of potash be mixed in the proportion to make a perfect interchange, some of the former will remain in the liquid, proportional to the quantity of the latter which escapes decomposition; whereas the absence both of sulphuric acid and baryta from the liquid can only occur, when the quantity of chloride of barium is insufficient for effecting complete double decomposition with the sulphate of potash. So that when the proportions appear to be right, they are certainly wrong; and they may be right, when they appear to be wrong. It is obvious, too, that Dr. Thomson's analysis of sulphate of potash by means of chloride of barium, is not more satisfactory than his analysis of chloride of barium by sulphate of

potash. The equivalent of potash, deduced from that analysis, cannot be relied on; and his proof of 40 being the exact equivalent of sulphuric acid is also liable to objection. But the error upon which Dr. Thomson has so unhappily fallen, has been also committed by other chemists. Every analysis of sulphate of potash, or of salts containing this alkali and sulphuric acid, must be regarded with suspicion. Thus the analysis of common alum by Dr. Thomson and Berzelius can scarcely be quite exact; and the analysis of potash-minerals, in which baryta has been separated by sulphuric acid, may also be suspected of slight inaccuracy.

The process by which I have endeavoured to analyze chloride of barium consists of two parts. In the first, a given quantity of the chloride was dissolved in water, and the baryta thrown down as sulphate by sulphuric acid. In the second, a similar solution was precipitated by nitrate of silver, and the chlorine inferred from the quantity of fused hornsilver which was produced. The quantity of chloride of barium employed in each experiment varied from 30 to 40 or 45 grains. The sulphuric acid had of course been purified by distillation, and left no residue when evaporated on platinum.

The process by sulphuric acid was varied: one while the solution and precipitate were evaporated to dryness in a platinum capsule; and at another, the insoluble sulphate was collected on a double filter. Both methods were frequently repeated, and the sulphate of baryta was always dried by exposure to a red heat. The quantity of sulphate of baryta obtained by the first method from 100 parts of the chloride ranged from 112.17 to 112.2, being more frequently the latter than the former; and 112.19 may be adopted as a mean of the most successful experiments. The quantity obtained by filtration fell rather short of this, varying in the best experiments from 112.08 to 112.12. The difference is referable to a trace of sulphate of baryta being retained by the acid solution, in which it may really be detected by evaporation. The first series of experiments may therefore be considered the more accurate, and it may be inferred that 100 parts of pure chloride of barium are capable of yielding 112.19 parts of sulphate of baryta. This result agrees very closely with that stated by Berzelius in the last edition of his System of Chemistry, who in one experiment got 112.17, and in another 112.18, of sulphate from 100 parts of chloride of barium. According to Dr. Thomson, 100 parts of the

chloride yield only 111.32 parts of sulphate of baryta. It is proper to state, in reference to the foregoing experiments, that traces of chloride of barium are apt to adhere to the sulphate of baryta; but this source of error is easily avoided by decanting the supernatant fluid after subsidence, and stirring the precipitate with hot water acidulated with sulphuric acid.

In order to determine the chlorine of chloride of barium by means of silver, it was desirable to ascertain the composition of hornsilver. For this purpose some fine silver containing only traces of gold and copper was dissolved in nitric acid, precipitated by sea-salt, digested in dilute nitro-muriatic acid, and washed. The dry chloride was then reduced by means of carbonate of potash in the usual manner, and after throwing a few fragments of nitre upon the fused metal, it was granulated and then boiled repeatedly in distilled water. In the silver thus prepared I could not detect potash, gold, copper, or any other impurity; whereas it is difficult in employing common silver, to purify it completely by one operation.

1. Of this silver 28.407 grains were dissolved in pure nitric, and precipitated by pure muriatic acid, both of which had been prepared with the greatest care. The whole mass was evaporated to dryness, and yielded 37.737 grains of fused chloride of silver.

2. In a second similar experiment 41.917 grains of silver yielded 55.678 grains of hornsilver.

3. In a third, 40.006 grains of silver yielded 53.143 of hornsilver.

According to the first and third experiments 100 parts of silver correspond to 132.84, and according to the second to 132.83 parts of hornsilver.

4. In a fourth experiment, 30.922 grains of silver were dissolved in nitric acid, and precipitated by muriate of baryta in excess. The precipitate after being carefully washed and collected on a double filter, yielded 41.07 grains of fused chloride; and hence the silver and chloride are in the ratio of 100 to 132.82.

5. In a fifth experiment, 42.255 grains of silver were dissolved as usual, precipitated by an excess of muriatic acid, and collected on a double filter. The fused chloride amounted to 56.09 grains, giving the proportion of 100 to 132.74. When the silver is thus precipitated by free muriatic acid, and the chloride collected on a filter, the result is constantly below that obtained by the other

methods, owing to a trace of the chloride being dissolved by the strong acid solution.

It may be inferred, as a mean of the four first experiments, that 100 parts of silver correspond to 132.83 parts of chloride of silver. The proportion stated by Berzelius is 100 to 132.75; and it is estimated at 100 to 132.72 by Dr. Thomson. All these results, therefore, are closely correspondent.

From one of the experiments (No. 4.) just mentioned, it is manifest that the precipitation of chloride of barium by nitrate of silver does not involve any appreciable source of error. To be quite certain, however, as to this fact, chloride of barium was mixed with nitrate of silver in excess, and the precipitate carefully washed. It was then boiled in distilled water, and the fluid examined for silver and baryta; but not a trace of either could be detected. It dissolved completely in ammonia, and the addition of sulphuric acid did not cause the slightest turbidity.

In five analyses made by precipitating chloride of barium by an excess of nitrate of silver, I obtained the following proportions.

	Chloride of Barium.	Chloride of Silver.
Exp. 1.	100 yielded	137.45
2.	100	137.54
3.	100	137.70
4.	100	137.62
5.	100	137.64

Though all these analyses were made with great care, the last two were the most successful, as being less influenced by errors of manipulation than the others. Instead, therefore, of taking the mean of the five, which is 100 to 137.61, I adopt the mean of the two last experiments, which is 100 to 137.63. In one of these the precipitate was washed with distilled water only, and in the other with water acidulated with nitric acid. Berzelius in his experiments on this subject found that 100 parts of chloride of barium corresponded to 138.06 in one experiment, and 138.08 in another. This is the only material difference between us which I have yet had occasion to notice. It induced me to reconsider every part of my experiments; but as I am unable to detect the slightest inaccuracy in the two analyses from which my result was derived, I cannot

hesitate to adopt it. I conclude, accordingly, that 100 parts of chloride of barium correspond to 137.63 parts of chloride of silver; and as, consistently with the preceding researches, this quantity of hornsilver contains 34.016 parts of chlorine, it follows that chloride of barium consists of

Barium	65.984
Chlorine	34.016
	100.000

Its constitution according to Dr. Thomson and Berzelius is shown by the following numbers:

	Thomson.	Berzelius.
Barium	66.037	65.926
Chlorine	33.963	34.074
	100.000	100.000

It is impracticable, from the composition of chloride of barium as above stated, to make any satisfactory inference relative to the real equivalent of barium, because the real equivalent of chlorine is not yet clearly ascertained. By Dr. Thomson it is estimated at 36, and by Berzelius at 35.43; and on calculating the equivalent of barium according to both estimates, the following result will be obtained.

Barium	69.832	68.726
Chlorine	36.000	35.430
	105.832	104.156

Hence if 36 is the equivalent of chlorine, that of barium will be 69.832, or very near 70 as stated by Dr. Thomson; but if the calculation be continued, still taking the results of my experiments as its basis, the equivalent of sulphuric acid will turn out to be 40.901 instead of 40. From these considerations it appears evident that at least one of the equivalent numbers concerned in the calculation must be incorrect. I abstain, however, from offering any further opinion on this point at present, as it will form the subject of another communication.

XXV. *On a new series of periodical colours produced by the grooved surfaces of metallic and transparent bodies.* By DAVID BREWSTER, *L.L.D. F.R.S. L. & E.*

Read May 21, 1829.

IN the year 1822, when I received from Mr. BARTON some very fine specimens of his Iris ornaments, I availed myself of the opportunity of performing a series of experiments on the action of grooved surfaces upon light. As the subject was to a certain extent new, many of the results which I obtained seemed to possess considerable interest, and I accordingly communicated to the Royal Society of Edinburgh a general account of them, which was read on the 3rd of February 1823. The interruptions, however, of professional pursuits prevented me, but at distant intervals, from pursuing the inquiry; and having found that M. FRAUNHOFER was actively engaged in the very same research, with all the advantages of the finest apparatus and materials, I abandoned the subject, though with some reluctance, to his superior powers and means of investigation. During a visit paid to Edinburgh by the Chevalier YELIN, a friend of FRAUNHOFER's and a distinguished member of the Academy of Sciences of Munich, I showed him the general results which I had obtained; and as he assured me that the phenomena which had principally occupied my attention had entirely escaped the notice of his friend*, I was thus induced to resume my labours, the results of which, in relation to one branch of the subject, I shall now submit to the consideration of the Society.

When a flat and polished metallic surface is covered with equal and equidistant grooves, we may characterize it by the relation of two quantities, one of which m represents the breadth of each groove, or of the surface that is removed, while the other n represents the breadth of the intermediate space, or

* The memoir of M. FRAUNHOFER was read to the Bavarian Academy of Sciences on the 14th of June 1823; and has no relation to the subject of this paper.

of the original surface that is left. If the image of a candle is seen by reflexion from such a surface, the trace of the plane of reflexion being parallel to the grooves, we observe the colourless image of a candle in the middle of a row of prismatic images arranged in a line perpendicular to the grooves. The colourless image of the candle is formed by the original portions n of the metallic surface, while the prismatic images are formed by the sides of the grooves m. This may be demonstrated ocularly by increasing m, and consequently diminishing n till the latter nearly disappears. In this case the intensity of the prismatic images rises to a maximum, while the ordinary colourless image becomes extremely faint, and vice versâ. The general phenomena of the prismatic images, such as their distance from the common image, and the dispersion of their colours, depend entirely on the magnitude of $m + n$, or the number of grooves and intervals that occupy any given space; and the laws of these phenomena have been accurately determined by M. Fraunhofer.

In the course of my examination of the prismatic images, I observed in some specimens an unaccountable defalcation of particular colours, varying with the angle of incidence, and sometimes affecting one of the images and not the others. It sometimes appeared in close and sometimes in wide systems of grooves, and from the symmetry of its effects, it became obvious that it was not owing to any accidental cause. In the specimen in which it was most distinctly seen, I was surprised to observe that the white image reflected from the original surface of the steel was itself slightly coloured; that its tint varied with the angle of incidence, and had some relation to the defalcation of colour in the prismatic images.

Hitherto I had used a small disc of light, but in order to observe through a great range of incidence I employed a long narrow rectangular aperture, which gave a convergent beam of 30° or 40°. I thus saw a series of very interesting phenomena. The ordinary image of the aperture, as formed by the spaces n, was crossed in a direction perpendicular to its length, with broad coloured fringes varying in their tints from 90° to 0° of incidence. This remarkable effect I observed in various specimens, having from 500 to 10,000 grooves in an inch. In a specimen with 1000 grooves in an inch, or in which

$m + n$ = 1000dth of an inch, no less than four complete orders of colours were developed as shown in the following Table.

	° ′		° ′
White	90 00	Bluish green	54 30
Yellow	80½	Yellowish green	53 15
Reddish orange	77½	Whitish green	51
Pink	76 20	Whitish yellow	49
Junction of pink and blue	75 40	Yellow	47 15
Brilliant blue	74 30	Pinkish yellow	41
Whitish	71	Pink red	36
Yellow	64 45	Whitish pink	31
Pink	59 45	Green	24
Junction of pink and blue	58 10	Yellow	10
Blue	56	Reddish	0

These colours are obviously those of the reflected rings in thin plates. By turning the steel plate round in azimuth, the very same colours are seen at the same angles of incidence, and they suffer no change either by varying the distance of the luminous aperture, or the distance of the eye of the observer.

I now examined various other specimens which possessed the same property. In some there were three orders of colours, in others two, and others one, while in some only one or two tints of the first order were developed. These different effects are more minutely detailed in the following Table.

Number of grooves in an inch.	Orders and portions of orders of colours developed from 90° up to 0° of incidence.
500	Citron yellow of the first order.
625	One complete order, and up to reddish yellow of the second order. Colours very dilute.
1000	Four complete orders of colours.
1000	One complete order, together with blue green and yellowish green of the second order.
1250	One complete order, together with blue and bluish green of the second order. Colours exceedingly faint and diluted.
2000	One complete order, together with blue green and greenish yellow of the second order.

Number of grooves in an inch.	Orders and portions of orders of colours developed from 90° up to 0° of incidence.
2000 on wax.	One complete order, together with greenish yellow of the second order.
2000 on wax.	One complete order, together with gamboge yellow of the second order.
2500 ——	One complete order, together with the full blue of the second order.
3333 ——	Gamboge yellow of the first order.
5000 ——	One complete order, together with bluish white of the second order. Colours more dilute than in No. 5.
10,000 ——	One complete order, together with blue and fainter blue of the second order.

It is obvious from the preceding Table that the diversity of effect produced by different specimens does not depend upon the quantity $m + n$, but upon n. The more that the original surface is ploughed away by the cutting diamond, the more brilliant were the tints, and the more numerous the orders of colours.

I was now desirous of seeing what effect would be produced when the original surface was almost wholly removed; and Mr. BARTON was so obliging as to execute for me a specimen containing 2000 grooves in an inch, in which this was nearly effected. His diamond point, however, having unfortunately broken before he had executed any considerable space, I was unable to make all the experiments with it which I could have wished.

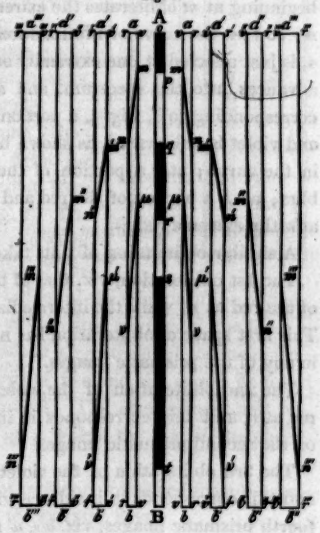
This specimen produced four complete orders of colours, all of which were developed at much greater angles of incidence than those in the preceding Tables.

	° ′		° ′
White	90 00	Red.	
Straw yellow.		Pink.	
Faint red.		Second limit of pink and blue	69 40
Pink.		Blue.	
First limit of pink and blue .	80 00	Green.	
Blue.		Yellowish green.	
Green.		Yellow.	
Yellow.		Orange.	

	° ′		° ′
Scarlet.		Brilliant green.	
Purple.		Yellowish green.	
Third limit of pink and blue .	48 00	Yellow.	
Blue.		Reddish	10 00

Such being the phenomena exhibited by the ordinary image formed by reflexion from the original spaces n, I now proceeded to examine the prismatic images in the first specimen with 1000 grooves, and I observed the following appearances.

Fig. 1.



Let **A B**, Fig. 1, be the reflected image of the rectangular aperture from the spaces n, and $a\ b$, $a'\ b'$, $a''\ b''$, $a'''\ b'''$, the prismatic images of it, $v\ v$, $v'\ v'$, &c. being the violet sides, and $r\ r$, $r'\ r'$, &c. the red sides of these spectra. Then in the

1st spectrum $a\ b$, the violet rays are obliterated at m at an incidence of 74°, and the red rays at n at an incidence of 66°, the intermediate colours, blue green, being obliterated at intermediate points between m and n, and at angles of incidence intermediate between 74° and 66°. In the

2nd spectrum $a'\ b'$, the violet rays are obliterated at m' at an incidence of 66° 20′, and the red at n' at 55° 45′. In the

3rd spectrum $a''\ b''$, the violet rays are obliterated at m'' at 57°, and the red at n'' at 41° 35′. And in the

4th spectrum $a'''\ b'''$, the violet rays are obliterated at m''' at 48°, and the red rays at n''' at 23° 30′.

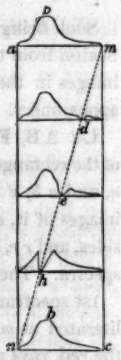
Another similar succession of obliterated tints takes place on all the prismatic images at a lesser incidence, as shown at $\mu\ \nu$, $\mu'\ \nu'$, the violet being obliterated at μ, and the red at ν, and the intermediate colours at intermediate

points. In this second succession the line $\mu\,\nu$ begins and ends at the same angle of incidence, as the line $m''\,n''$ in the third prismatic image $a''\,b'$; and the line $\mu'\,\nu'$ on the second prismatic image corresponds with $m'''\,n'''$ on the fourth prismatic image.

This singular obliteration of the colours is shown more clearly in Fig. 3, where $r\,m\,v\,n$ is a part of one of the prismatic images, $r\,v$ the red space, $g\,g$ the green space, $b\,b$ the blue, and $v\,v$ the violet space. The line of obliteration $m\,n$ in beginning at m obliterates the extreme violet at m; so that the curve of illumination $a\,b\,m$, Fig. 4, is just affected at one extremity m. The line advances into the spectrum, and at the point corresponding to d, Fig. 4, a portion of the blue and violet is obliterated, as shown by the notch in the curve; at e a portion of the green and blue; at h a portion of the red and green, and at n the extreme red.

Fig. 3.

Fig. 4.



A similar obliteration of tints takes place on the ordinary image A B.

The 1st obliteration, viz. that of the violet, takes place at o, Fig. 1, and that of the red at p; while the intermediate colours disappear at intermediate points. This first space of obliteration has no corresponding one at the same incidence in any of the prismatic images.

The 2nd obliteration of the violet in A B takes place at q, and that of the red at r, and this corresponds in incidence with the obliterations $m'\,n'$, $m'\,n'$ on the second prismatic image.

The 3rd obliteration of the violet takes place at s, and that of the red at t, and this corresponds in incidence with the four obliterations on the second and fourth prismatic images, viz. $\mu\,\nu$, $\mu'\,\nu'$, $m'''\,n'''$, $m'''\,n'''$.

In all these phenomena the points m, n, μ, ν, &c., are only the points of minimum intensity, or of maximum obliteration; for the tints never entirely disappear, and those obliterated at each line $m\,n$ form an oblique spectrum containing all the prismatic colours.

Fig. 2.

The analysis of these curious and apparently complicated phenomena becomes very simple when they are examined under homogeneous illumination. The effect produced in red light is represented in Fig. 2, where A B is the image of the rectangular aperture reflected from the faces *n* of the steel, and the four images on each side of it correspond with the prismatic images. All these nine images, however, consist of homogeneous red light, which is obliterated at the fifteen shaded rectangles, which are the minima of the new series of periodical colours which cross both the ordinary and the prismatic images. The centres *p*, *r*, *t*, *n*, *ν*, &c. of these rectangles correspond with the points marked with the same letters in Fig. 1; and if we had drawn the same figure for violet light, the centres of the rectangles would have corresponded with *o*, *q*, *s*, *m*, *μ*, &c. in Fig. 1. The rectangles should have been shaded off to represent the phenomena accurately, but the only object of the figure is to show to the eye the position and relations of the minima of the periods.

If it should be practicable to remove a still greater portion of the faces *n*, the first minimum *p*, Fig. 2, would commence at a greater angle of incidence; and other two rows of minima, namely, rows of five and six, would be found extending to the fifth and sixth prismatic images. The arrangement and succession of these is easily deducible from Fig. 2, where the law of the phenomenon is obvious to the eye.

The following table contains the angles of incidence reckoned from the perpendicular at which these minima occur in the extreme rays.

Position of the minima in red light.

	Ord. Im.	1st Prism. Im.	2nd Prism. Im.	3rd Prism. Im.	4th Prism. Im.
First minima *p*	76 0	66 0	55 45	41 35	23 30
Second minima *r* ..	55 45	41 35	23 30		
Third minima	23 30				

Position of the minima in violet light.

	Ord. Im.	1st Prism. Im.	2nd Prism. Im.	3rd Prism. Im.	4th Prism. Im.
First minima	81° 30′	74°	66° 20′	57°	48°
Second minima....	66 20	57	48		
Third minima.....	48				

When the steel with 1000 grooves is exposed to common light, and the incident ray is very near the perpendicular, the 5th, 6th, 7th, and 8th prismatic images are combined into a mass of whitish light terminated externally by a black space. As the angle of incidence increases, the 6th, 7th, 8th, and 9th images are combined into this mass, then the 7th, 8th, 9th, and 10th images, and so on, the black space which terminates this mass receding from the axis or image A B, Fig. 1, as the obliquity of the incident ray increases.

Having covered the steel plate with water and oil of cassia in succession, I found the angular distances of the black space to be as follows at the same incidence.

Air	12° 23′
Water	17 15
Oil of cassia	21 22

The sines of which are inversely as the indices of refraction of the fluids.

Phenomena analogous to those above described take place on the grooved surfaces of gold, silver, and calcareous spar, &c.

In order to study this subject under a more general aspect, I was desirous of examining the phenomena exhibited by grooved surfaces of different refractive powers. It was obviously impossible to procure systems of lines upon transparent bodies in which the grooves should have exactly the same distance and magnitude; but I conceived it practicable to impress upon different substances the very grooves which produced the preceding phenomena, and I succeeded in impressing the system of 1000 grooves upon tin, realgar, and isinglass.

The following results were obtained with Tin, the colours being those upon A B, Fig. 1.

White	90° 0′	1st junction of pink and blue	76° 20′
Yellow.		Greenish blue.	
Pink.		Yellow.	

	° ′		
Pink.		Yellow.	
2nd junction of pink and blue	57 40	Orange. Pink.	
Bluish green.		3rd junction of pink and blue.	

First minimum of red . . . . . 76°
Second ——— 61

The following results were obtained with Realgar.

	° ′		°
White	90 0	Yellow	63
Yellow	80	Bright pink	54
Pink	75 30	2nd junction of pink and blue	47
1st junction of pink and blue	73 10	Bluish green	41
Blue	72	Yellow	36
Bluish green	70 15	Pink	32
		More and more pink.	

First minimum of red . . . . . 72° 0′
Second ——— 61 15

The following results were obtained with Isinglass. The colours were generally the same as in the steel.

The first limit of pink and blue was at . . . . . 75° 45′
The blue of second order 73 45
The second limit of pink and blue was at 54 30

In these experiments the tin gave nearly the same results as the steel; but in the realgar and the isinglass similar tints were produced at a less angle of incidence than in the steel. The minima of the periods were exhibited very finely on the isinglass, and were produced at smaller angles of incidence.

In a specimen with 1000 grooves upon isinglass, the third pink, or that seen upon steel at 36°, was the highest; but after drying, the pink descended to yellow, and subsequently to green.

If the isinglass is removed from the steel when it is still soft, the edges of the grooves get rounded and lose their sharpness, and only one prismatic image is seen on each side of the ordinary image, as in mother-of-pearl.

The mass of white light is finely seen in the impressions taken upon tin, but never appears upon isinglass.

The preceding experiments do not afford any precise data for determining the influences of refractive power. The realgar and the isinglass give fewer periods of colour so as to indicate that, cæteris paribus, a diminution of refractive power, produces a diminution in the number and orders of colours, or causes the minima to be developed at a less incidence. This indication, however, is opposed by the fact, that as the isinglass dries and consequently increases in refractive power, the periods diminish in number, and the minima are produced at less incidences. The modification of the tints by a change of refractive power is here masked by the influence of other causes, namely, an inferiority in the sharpness of the impression to that of the original surface, and a rounding of the narrow spaces *n* subsequently produced by induration. In the specimen of isinglass, therefore, already mentioned, which gave the first limit of pink and blue at nearly the same angle as the steel, it is probable that it would have developed the same limit at a greater inclination had the impression been as sharp as the original.

In this uncertainty I conceived that the influence of a variable refractive power would be best obtained by placing different fluids on the surface of the grooved steel; and upon using alcohol and oil of cassia my expectations were fulfilled.

The following were the results:

Number of grooves in an inch.	Maximum tint without a fluid.	Maximum tint, with water, alcohol, and oil of cassia.
312	No colour	1. Water. Tinge of yellow. 2. Alcohol. Tinge of yellow. 3. Oil of cassia. Faint reddish yellow.
500	Citron yellow of first order	1. Water. Tinge of red. 2. Alcohol. Diluted pink. 3. Oil of cassia. A bluer pink.
625	Reddish yellow of second order	1. Water. Faint pink of second order. 2. Alcohol. Ditto more pink. 3. Oil of cassia. Bluish pink of second order.
1000	Yellowish green of second order	1. Water. Pinkish red, second order. 2. Alcohol. Brilliant pink, ditto. 3. Oil of cassia. Greenish blue, third order.
1250	Bluish green faint	1. Water. Yellow of second order. 2. Alcohol. Yellower. 3. Oil of cassia. Yellowish pink.

Number of grooves in an inch.	Maximum tint without a fluid.	Maximum tint, with water, alcohol, and oil of cassia.
2000	Greenish yellow of second order	1. Water. Brownish red, second order. 2. Alcohol. Pinkish red, ditto. 3. Oil of cassia. Greenish blue.
2500	Blue, second order	1. Water. Dilute green. 2. Alcohol. Greenish white, second order. 3. Oil of cassia. Bright gamboge yellow.
3333	Gamboge yellow of first order	1. Water. Pinkish red, first order. 2. Alcohol. Reddish pink. 3. Oil of cassia. Bright blue, second order.
5000	Bluish white of second order	1. Water. Pale yellow. 2. Alcohol. Yellow with tinge of orange. 3. Oil of cassia. Yellowish pink, second order.
10,000	Fine blue of second order.....	1. Water. Greenish white of second order. 2. Alcohol. Yellowish white. 3. Oil of cassia. Brilliant gamboge yellow.

I obtained similar results with grooves impressed upon wax; so that we may now safely draw the conclusion that more orders of colours, and consequently higher tints at a given incidence, are developed by diminishing the refractive power of the grooved surface.

The influence of refractive power on the tints of the ordinary image being thus determined, it became interesting to ascertain its effects on the obliterated tints of the prismatic images. As these tints never appeared unless when that of the ordinary image exceeded the blue of the second order, I took the specimen with 10,000 grooves, which had for its maximum tint a blue of the second order, but which exhibited no obliterated tints in the prismatic images. Having placed upon it a film of oil of cassia, I raised the blue to a gamboge yellow, and I found that the fluid developed the phenomena of obliterated tints on the first prismatic image. Owing to the great breadth of the spectrum, the distinct separation of the colours which composed it, and the great length of the line of obliteration, this phenomenon was one of the most beautiful and remarkable that I have ever witnessed.

Hitherto I had examined the minima in the prismatic images as symmetrically related in position to the minima in the ordinary image, as shown in Figs. 1 and 2; but in studying some specimens in which the spaces n were very broad, and the grooves or spaces m comparatively narrow, I was surprised to observe obliterated tints on the prismatic images, while the ordinary

image was entirely free of colour. This took place in two specimens, one of which had 312, and the other 625 grooves in an inch. The spaces n were here far too wide to produce the new tints, and so were the spaces m; but upon applying the microscope to the grooves m, I saw that they were formed by two or more grooves ploughed out by the cutting point; so that each space m actually consisted of smaller reflecting spaces, which were sufficiently minute to produce the periodical colours.

Although in these specimens, therefore, when m is nearly equal to n, we observe a beautiful coincidence between the positions of the minima on the ordinary and on the prismatic images, yet the fact above described seems to show that they are separate phenomena, and depend, when the grooves are single, on the relation between m and n.

The preceding observations relate solely to rays reflected from grooved surfaces; but in consequence of the almost perfect transparency of isinglass in thin plates, I have been enabled to examine the transmitted tints. The colours which are thus seen on the ordinary image are extremely brilliant, but they seem to have no relation whatever, either in number or in quality, to the reflected tints. In the specimen which gave by reflexion three orders of colours, those seen by transmission were only the following.

Fine blue	85° of incidence.
Purple.	
Red.	
Orange.	
Yellow	0 vertical incidence.

Another specimen from the same steel plate gave, when soft and newly taken off, a bright purple at a perpendicular incidence, which passed through pink and blue at greater incidences. But in the process of induration, the vertical purple became red, orange and yellow. In a third impression the perpendicular tint was a bright pink when soft, which descended to yellow when drier.

In order to observe the relation between the reflected and transmitted tints, I took a fresh impression on very transparent isinglass, and obtained the following results.

Reflected tints.	Transmitted tints.	Angles of incidence.
Yellow	Deep blue	90
Orange	Paler blue.	
Pink	Blue.	
First limit of pink and blue . . .	Blue.	
Blue	Pink.	
Green	Orange pink.	
Yellow	Orange.	
Orange	Yellow.	
Pink	Yellow.	
Second limit of pink and blue .	Yellow.	
Blue	Yellow	0

The comparison of these tints affords the most satisfactory evidence that they are not complementary to each other. The transmitted tints of the ordinary prismatic images always increase in brightness as the angle of incidence diminishes, while the reflected tints become fainter.

As I had preserved the different specimens of isinglass with which these experiments were made, it became interesting to observe the changes which their colours had undergone after a lapse of six years. The following was the result.

1. A specimen with 1000 grooves exhibited no colours on the ordinary image either by reflexion or transmission. The prismatic images of a candle were very faint, and the 4th could scarcely be seen.

2. Another specimen of 1000 grooves gave by reflexion one period of colours from white at great incidences through yellow up to purple at a vertical incidence. By transmission a little yellow only was seen at a great incidence.

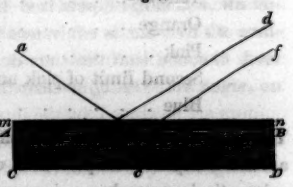
3. A third specimen of 1000 grooves which had been a fine sharp impression, gave by reflexion two orders of colours, the first limit of pink and blue being at 57° 45′, and the second limit nearly at a vertical incidence, a deep pink appearing at 10°. By transmission the isinglass gave a bluish green at the greatest incidence which passed at lesser incidences through purple to yellow, which was the maximum tint.

In all these specimens the colours remain the same in all azimuths, provided the angle of incidence is invariable.

As the steel plate from which all these impressions had been taken was much injured, I resolved to grind down its surface by a polishing powder, and to observe the changes which took place. As the effect of this was to increase the spaces *n*, the colours on the ordinary image soon disappeared. The phenomenon of the obliterated tints was no longer seen, the mass of white light disappeared, and from the rounding of the edges of the grooves the prismatic images were fewer in number, though their distance was unchanged.

When one of the impressed films of isinglass *m n*, Fig. 5, was laid upon a plate of glass A B C D, and was in optical contact with it, a series of fringes was seen across the images reflected from the second surface C D of the glass. These fringes seen by the eye at *d f*, and formed by the rays *a b c e f*, are parallel to the grooves on the isinglass, and their breadth diminishes as the thickness A C of the glass is increased. When the grooves were 1000 in an inch, these fringes were nearly as distinct as the prismatic images, one fringe appearing to bisect each image when the thickness A C was about $\frac{1}{10}$th of an inch. They were much more numerous, and even crossed the principal image when A C was $\frac{1}{4}$th of an inch; but when A C was $\frac{1}{2}$th of an inch, no fringes were seen across the second image.

Fig. 5.



These fringes have the same origin as those which I have described in the Edinburgh Transactions. In the first specimen, where A C was $\frac{1}{10}$th of an inch, its two surfaces were not parallel, and the direction of the grooves in the isinglass was accidentally perpendicular to the common section of the two surfaces of the glass. Hence the fringes produced by the glass were parallel to the prismatic images from the isinglass. But when the specimen is turned round, the isinglass fringes reflected from the back of the glass are crossed by those produced by the glass, giving to the former the appearance of a coloured rope, in which the coils pass along the longitudinal spectra with singular beauty.

Such are the leading phenomena of this new and remarkable class of periodical colours; but though their general law and the circumstances upon which they depend seem to be pretty clearly shown in the preceding experiments, yet

I feel great difficulty in assigning a satisfactory cause for their production. That they are not owing to the diffraction and interference of the rays reflected from two or more of the surfaces *n*, considered as narrow slits or apertures, is obvious; for in that case they would be affected by the distance of the luminous object and the distance of the eye, and the colours would form bands parallel to the direction of the grooves.

In my experiments on the production of the complementary colours by the metallic reflexion of polarized light, I have shown that one reflexion from a plate of silver, &c. is equivalent in its action to a given thickness of a crystallized film, and that the tints descend in the scale by increasing the angle of incidence as if the equivalent film had diminished in thickness. That these colours are produced by the interference of two pencils, one of which suffers reflexion later than the other, cannot be doubted; but whether these two portions are reflected within the sphere of reflecting activity, at such distances as to produce colours by their interference, or whether the one is reflected in the usual manner, while the other is not reflected till it has penetrated a certain thickness of the polished metal, it is not easy to ascertain.

If either of these effects takes place with polarized light, an analogous effect should be produced with common light, though the intensity of the interfering pencils might in this case be very inconsiderable.

If we suppose that the spaces *n* are smaller than the distance to which the reflecting force extends, the removal of the metal from the adjacent grooves must diminish the reflecting force of these spaces. That this is the case may, we think, be inferred from direct experiment. At the separating surface of the steel and a fluid, we observe a certain change in the action of the steel surface, which can be ascribed to no other cause than the diminution of the refractive and reflective power of the surface. Now it is manifest from experiment that the diminution of the spaces *n* has exactly the same effect, the colours not only being rendered brighter by each of these causes, but the minima being produced at greater angles of incidence.

Since in a system of grooves with only 312 in an inch, oil of cassia developes colours which did not previously exist, it is evident that if we had fluids of much higher refractive power, colours would be produced when the spaces *n* were much larger, and when the fluid approached in refractive density to that

of the metal, we should witness the periodical colours without any grooves at all on the reflecting surface; so that the phenomena would then become identical with those which are developed at the separating surface of transparent bodies.

We can scarcely, therefore, avoid the conclusion, that the removal of the substance from the grooves, whether they are made on metal or on transparent bodies, diminishes the refractive power of the intermediate spaces. On the hypothesis of emission, this abstraction of the reflecting matter may be regarded as equivalent to a diminution of the density of the surface; while on the undulatory hypothesis, the effect may be ascribed to the condition of the ether arising from a variation in its density or elasticity towards the extremities of a number of salient points.

XXVI. *On the Nerves of the Face; being a second paper on that subject. By* CHARLES BELL, *Esq. Fellow of the Royal Society.*

Read May 28, 1829.

I HAVE to beg the indulgence of the Society to some minute details of anatomy, for the sake of those deductions which can be attained by no other means: and that a zeal for its cultivation may be preserved among us. There is an obvious practical benefit derived from anatomy, but the public do not comprehend its importance as a science. It is to the Royal Society that those who prosecute this science must look for countenance in their slow and painful investigations.

Nine years ago, at the request of our late President, I submitted to the Society a paper on the Nervous System; in which I arranged the nerves strictly according to the anatomy, and illustrated the principles of the arrangement, by exhibiting the different functions of the Nerves of the Face. On presenting a second paper on the same part of the nervous system after so considerable a lapse of time, there will be some novelty both in the facts and in the illustrations; yet I have more gratification in showing that after the most minute inquiries in different countries, my positions drawn from the anatomy have been admitted, and my reasoning on the experiments, with one exception, found to be correct. Confident in the accuracy of my deductions from the anatomy of the fifth nerve, I had attributed to one of its branches a function which belongs to another branch of the same nerve. The subject will form a part of the present paper.

After the announcement of the facts in my first paper, the inquiry became interesting from its application to medical practice. I must take another opportunity of thanking those gentlemen who have so liberally afforded additional proofs of the truth of my principles. I must restrict myself in refer-

ring to them here, since I am desirous that the Society's Transactions should contain only the philosophical part of the inquiry.

The system of Willis, of which we have an elegant account in the posthumous works of Dr. Baillie, prevailed universally in the schools when I entered on these inquiries. In opposition to that system I demonstrated that the nerves hitherto supposed to possess the same powers, consisted of filaments having different roots, and performing different functions. I found myself embarked in this investigation, from observing the course which the nerves took in their distribution through the body. Conceiving that the devious course and reunion of the nerves were for a purpose, I sought in their origins for the cause of their seeming irregularity. It was discovered that the roots of the nerves arose from distinct columns of nervous matter, and that on these columns depended their different properties. Those which were called the common nerves, that is, the nerves which arise from the spinal marrow, thirty in number, were found to consist each of two nerves derived from distinct columns, one for sensation and one for motion. In the further pursuit of this subject, there was reason to conclude that the spinal marrow contained not only the columns for bestowing sensation and motion, but also another column, the office of which was to combine the actions of respiration. I then drew the attention of the Society to the course of the fifth nerve of the brain according to Willis. I showed that it had the same double root as the spinal nerves, that it had a ganglion, and that part of the nerve passed free of the ganglion; and that from all these points of resemblance, it was to be considered as the anterior or superior of the spinal nerves, of that system which is called symmetrical, and which ministers to the same functions in all classes of animals, bestowing sensibility and the locomotive powers, but deficient in those filaments which command the respiratory motions. I am particular in restating this, because from time to time it has been reported that I had abandoned my original opinions; whereas every thing has tended to confirm them.

From the general view of the nervous system, I drew attention to the superadded or irregular nerves. Having shown that the original or symmetrical system of nerves, of which the fifth was one, had no power over the motions of respiration, and that the human countenance in all its motions, with the exception of mastication, bore relation to the actions of respiration, it was

therefore required that another nerve besides the fifth, should be sent to the face. Having shown also that the roots of the fifth nerve were distant from that column of nervous matter which gives origin to the nerves of the respiratory system, and that it could not therefore minister to the motions of the face which are connected with respiration; and that another nerve, the portio dura, having its root in common with the nerves of respiration, took its course to the face,—the subject was prepared for experiment.

By experiments on the nerves of the face these three things were proved: First, that the sensibility of the head and face depended on the fifth pair of nerves. Secondly, that the muscular branches of the fifth were for mastication: and in the Third place, it was proved that the portio dura of the seventh, or respiratory nerve of the face, controuled the motions of the features, performing all those motions, voluntary or involuntary, which are necessarily connected with respiration;—such as breathing, sucking, swallowing, and speaking, with all the varieties of expression.

Reserving the details, I shall now state shortly the occurrences which I have witnessed since the publication of that paper; as they afford convincing proofs of the correctness of these opinions.

The first instance was in a man shot with a pistol ball, which entered the ear and tore across the portio dura at its root. All motion on the same side of the face from that time ceased; but he continued in possession of the sensibility of the integuments of that side of the face.

The next instance was in a man wounded by the horn of an ox. The point of the horn entered under the angle of the jaw and came out before the ear, tearing across the portio dura. He remains now a singular proof of the effects of the loss of function in the muscles of the face by this nerve being divided. The forehead of the corresponding side is without motion, the eyelids remain open, the nostril has no motion in breathing, and the mouth is drawn to the opposite side. The muscles of the face, by long disuse are degenerated, and the integuments of the wounded side of the face are become like a membrane stretched over the skull. They have lost their firmness, and the flesh under them is wasted, with the exception of certain muscles, the reason of which will be understood on perusing the anatomical description in the present paper. In this man the sensibility of the face is perfect. The same nerve (portio dura)

has been divided in the extirpation of a tumour from before the ear, and the immediate effect has been horrible distortion of the face by the prevalence of the muscles of the opposite side, but without the loss of sensibility; and that distortion is unhappily increased when a pleasurable emotion should be reflected in the countenance.

These facts are so distinct, that I cannot presume to detain the Society with the instances of the lesser defects which I have witnessed from the more partial injuries or temporary diseases of the nerve;—such as distortion of the features produced by glands pressing on this nerve, paralysis from suppurations in the ear affecting the nerve in its passage, or temporary derangement disturbing one or more of its functions.

As to the fifth nerve, the facts are equally impressive, and correspond with our former experiments and opinions. By a small sacculated tumour affecting the roots of this nerve, the sensibility was destroyed in all the parts supplied by its widely extended branches; that is, in all the side of the head and face and the side of the tongue, whilst the motion of the face remained. Two circumstances affecting this nerve have occurred with most curious coincidence in the symptoms. By the drawing of a tooth from the lower jaw, the nerve which comes out upon the chin to supply one half of the lip was injured, and exactly this half of the lip was rendered insensible. When the patient put his mouth to a tumbler he thought they had given him a broken glass! Precisely the same thing occurred from the division of that branch of the fifth nerve, which goes to one half of the upper lip. A gentleman falling, a sharp point entered his cheek and divided the infra orbitary nerve: the effect was loss of sensation without loss of motion, in that half of the upper lip to which the nerve is distributed. The remarkable circumstance was, that this individual made the same remark when the cup was put to his lip:—that they had given him a broken one! The part of the cup which was placed in contact with the insensible portion of the lip appeared to him to be broken off.

I have had two or three instances before me of disease affecting the ophthalmic branch of the fifth nerve, and producing total insensibility of the eye and eyelids, without loss of vision; whilst the eyelids continued to be closed and the eyebrow to be moved by the influence of the portio dura of the seventh nerve.

Such are a few of the facts which have been reaped from a patient reliance on the correctness of my first deductions, and I would now urge them in proof of the importance of reasoning upon the anatomy. All these nerves have been repeatedly divided, by almost every surgeon of eminence in the three kingdoms. Although some have performed the operation of dividing the nerves frequently, and one eminent gentleman had done it six times on the face of the same man, all these operations have been performed without giving rise to the suspicion that these nerves bestowed different properties. Even now, so slow is the progress of improvement, it is stated by a surgeon that he will not hesitate to cut the portio dura in the case of tic douloureux. My duty is performed when I give publicity to the facts which prove that horrible distortion of the whole countenance, the loss of distinct articulation, the loss of expression, the loss of motion of the eyelids, and consequent inflammation of the eye, must follow such an operation.

Much has been said in favour of experiments when made by men who are positively without any expectation of the result, or, as they affirm, are unbiassed. The only instances of this that I can allow, are when the surgeon cuts the nerves of the face in a surgical operation. In such operations as these for tic douloureux, he is indeed unbiassed; and we have seen the result, that after fifty years of such experience we remained quite ignorant of the distinctions in these nerves. But on the other hand when attention is roused to inquiry by aantomy, facts are obtained of the utmost importance both to the knowledge of disease and to the safe practice of surgery.

Of the Motor or Manducatory portion of the Fifth Nerve.

The fifth nerve is usually called Trigeminus, from piercing the skull in three grand divisions. But when it has been shown that it is composed of two distinct roots having different functions, the accidental circumstance of its divisions passing through the bones yields in importance to another inquiry, How is the muscular portion of the nerve distributed?

Since the publication of my first paper this inquiry has assumed importance; although the principal facts of the anatomy were known to Wrisberg, Santorini, Paletta, Prochaska, and Sœmmerring. But in no author is the ana-

tomy of the motor portion of the nerve traced with sufficient minuteness, or regard to the distinct uses of the muscular and sensitive divisions.

The motor division of the fifth nerve passes under the Gasserian ganglion, and free of it. It is not seen when we look from above, as in the plates of Monro. When the nerve is turned up and dissected, this portion is seen to form about a fifth part of the whole nerve. It is tied to the larger portion before advancing to the ganglion, by filaments which have been sometimes taken for nerves.

Having passed the ganglion, it attaches itself slightly to the superior maxillary nerve, but this is apparently a membranous connection only*. The nerve itself joins the third grand division after passing the foramen ovale. At this point the muscular and sensitive portions of the nerves are matted together, and form a mass which between the fingers feels like a knot†. There is, however, no red and fleshy-like matter interposed here, as in the Gasserian ganglion of the trunk of the nerve. But the filaments of both portions of the nerve are here so complexly and intimately combined, that all the branches which go off after this union are compound nerves, and have motor filaments in their composition.

It is, however, equally obvious that the gustatory division of the nerve which descends from this mass, has not the muscular portion given to it in that abundance which those branches have which take their course to the muscles of the jaws. The mandibulo-labralis, which also descends from this plexus, lies nearer the motor portion, and has a more distinct addition given to it than the gustatory nerve.

This motor or muscular portion which we are tracing, sends off no branch either in its course under the great ganglion, or after passing it about half an inch. But when it has arrived at the point of union with the ganglionic portion, the filaments become interwoven; and from this place the nerves are

* Gerard, commenting on Santorini, says that the anterior root (the motor) does give filaments to the superior maxillary division of the fifth. Prochaska (de Structura Nervorum) gives two views, Tab. ii. fig. v. vi. which represent an actual union of the anterior root and the superior maxillary nerve. In the plate, however, the twigs seem rather to go from the ganglionic into the motor division.

† Santorini says, it is a plexus like a ganglion, "in plexum vere ganglioformem mutatur."

compound, and go off diverging to their destinations. First, there are sent off nerves to the temporal, masseter, and pterygoid, muscles, also to the buccinator muscle. The temporal muscle receives a large and appropriate nerve. The nerve to the masseter passes between the coronoid and condyloid processes of the lower jawbone; but before going into the muscle it sends branches to the temporal muscle. The pterygoid muscles have each their appropriate nerves coming direct from this plexus.

Ramus Buccinalis Labialis.

This is a remarkable branch which arises from the same source, and goes to the cheek and lips. This nerve where it lies on the external pterygoid muscle sends one more branch to the temporal muscle; it then divides, one branch enters the buccinator muscle, and another is prolonged forwards. The division to the buccinator muscle is tortuous, which is no doubt a provision for its being undisturbed by the free motion of the cheek; its minute branches may be traced until lost among the muscular fibres, whilst others penetrate to the lining of the cheek. The prolonged branch is the labial division; it runs nearer the alveolar processes of the lower jaw, and becomes so superficial as to admit a union with the portio dura: from thence passing under the facial artery it may be traced into the triangularis or depressor anguli oris, the levator labiorum communis, and the lateral portion of the orbicularis oris.

In the distribution of the buccinalis labialis to the muscles of the mouth, it is joined, as I have said, by branches of the portio dura; and nothing is more striking than the manner in which this latter nerve passes over the masseter, a muscle of the jaw, to be profusely given to the muscles of the lips.

There is one more branch important to the physiology of the fifth nerve. At the root of the mandibulo-labralis (where it is sent off from the junction of the muscular and ganglionic portions), a small nerve takes its origin. This branch runs parallel to the greater nerve till it enters the foramen in the lower jaw; here it seems to enter, but does not; it takes a course on the inside of the jaw to arrive at its final destination, the mylo-hyoideus and the anterior belly of the digastricus, that is, to those muscles which open the mouth by drawing down the jaw.

We may for a moment interrupt our particular inquiry, to notice that all muscular nerves, and consequently the muscular divisions of the fifth nerve, form a plexus. The plexus, formed by the motor and ganglionic divisions of the fifth nerve before they diverge to the muscles of the lower jaw, corresponds with the plexus formed on the nerves sent to other classes of muscles. Even that branch of the third division of the fifth nerve which comes out before the ear, joins the portio dura in a plexus*; and this is the reason of that sensibility evinced in the facial nerve in making experiments upon it.

The form of the fifth nerve, and its resemblance to the spinal nerves, had struck some of the best continental anatomists. But as they had made no distinctions in the functions of the roots of the spinal nerves, so neither did they imagine any difference in the roots of the fifth nerve, and therefore no consequence resulted from having observed this resemblance. This part of the anatomy, together with the whole minute relations of the nerves, was a dead letter, and led to no inference.

But now resuming the course I have hitherto followed, the anatomy of the fifth nerve points to curious results. We see that the motor division of this nerve goes first to the muscles which close the jaw and give it the lateral or grinding motions. Secondly, we see that it is distributed to the muscles of the cheek, which place the morsel under the operation of the teeth; and thirdly, we find it going to the muscles which open the jaws.

We proceed to the second method of proof, by experiment. Does the fifth nerve move the jaw? is it indeed the manducatory nerve as suggested by the anatomy? Let the following experiments determine the fact.

Experiment I.

The root of the fifth nerve being exposed in an ass and irritated, the jaws closed with a snap.

Experiment II.

The fifth pair being divided in an ass, the jaw fell relaxed and powerless.

If we consider the action of mastication, we shall see what the consequence would be, were there no accordance between the motions of the lower jaw and

* See the adjoined plate.

the cheeks. Conceiving that there must be such an accordance, and contemplating the roots of the fifth pair and their distinct functions, I had imagined that this office was performed by the branches of the second division of the fifth. But finding that the connection between the motor root and the superior maxillary nerve proved to be only by cellular texture, and considering the affirmation of M. Magendie and those who followed him, that the infra-orbitary branch had no influence upon the lips, I prosecuted with more interest the Ramus Buccinalis Labialis. And nobody, I presume, will doubt that the distribution of this division confirms the notions drawn from the anatomy of the trunk,—not only that the fifth nerve is the manducatory nerve as belongs to the muscles of the jaws, but also that it is distributed to the muscles of the cheek and lips to bring them into correspondence with the motions of the jaws. Let us take in illustration the articulation of the bones. In the joints the muscles are attached to the capsular membrane in such a manner as to draw it from between the bones and adapt it to the degree of flexion of the joint. If the cheek were a passive membrane like the capsule of a joint, it would have required some such mechanical connection with the jaw or its muscles, as might have drawn it from between the teeth in the motions of mastication. But being a muscular part, to bring it into just relation with the motions of the teeth, it must have an accordance through nerves, and act in sympathy;—relax when the jaws are apart, and contract when they are closed. I think therefore we may perceive why a branch of the motor nerve of the muscles of the jaws sends a division to the muscles of the cheek and to the angle of the mouth.

By such a process of reasoning we see also why a branch of the same nerve should prolong its course under the chin to the muscles which are opponents to those which close the jaw.

In short, the motor portion of the fifth nerve sends no twigs with the ophthalmic division, nor the superior maxillary nerve, but only with the lower maxillary nerve. To the muscles of the lower jaw alone which are in action during mastication, and to the muscles necessarily associated in that action, the manducatory nerve is distributed.

It remains only that we observe what takes place in man, and compare the circumstances with experiments on brutes.

I was consulted in the case of a lady with an uncommon disease in the side of the head: the description of her condition puzzled me very much; there was so much said of tumours with pulsation on the head and face. But when I saw and examined her, the mystery disappeared; she had powerful spasms of the temporal and masseter muscles, which rose and swelled, under the excitement of a disease of the cheek, and with a pressure of the jaws so powerful as to displace the teeth. During this violent spasm of the muscles supplied by the fifth nerve, the motions of the features were free and unconstrained under the influence of the portio dura of the seventh nerve.

I have the precise counter-part to this morbid condition of the muscles of mastication in the case of a poor man now under my care. He has a disease affecting the fifth nerve of the left side, attended with the loss of sensibility of the side of the face and of the surfaces of the eye. In him there is no motion of the muscles of the jaw of the affected side. In chewing, the action is only on the right side of the head; the masseter muscle and temporal muscle of the left side do not rise or bulge out as in their natural actions; but his command over his features is perfect through the operation of the portio dura. It appears, therefore, that the disease of the fifth nerve, which has destroyed the sensibility on one side of the face, has caused a loss of motion in the muscles of the jaw on the same side.

A more frequent occurrence establishing the distinction of motions influenced by the fifth and seventh nerves, is presented in the case of paralysis of the portio dura; for then all the muscles waste but those supplied by the fifth. In the case referred to, of the man wounded by the horn of an ox, in whom the portio dura was torn, and who had the skin of his forehead, side of the nose, cheek and lips, deprived of all fleshiness and substance, and in fact wasted to mere skin, the muscles of the jaw were entire and prominent; and on introducing the finger into the mouth and making him imitate the motions of mastication, a weak contraction could be felt in the cheek*.

These facts close the evidence of the fifth nerve being a double nerve; not only the nerve of sensibility to the head and face, but a muscular nerve to the muscles of the jaws, active in mastication, and otherwise useful in all animals

* How often a question has occurred as to this motion in the cheeks, may be seen on referring to cases, p. 123, Exposition, &c. and p. 57, Appendix, 1st edition.

whose jaws are prehensile and used as hands. This curious fact, originally drawn from the anatomy and now confirmed by it, had nearly been obscured by experiment; since the external branches of the fifth nerve, those most exposed to the experimenter, are not muscular.

I am bound to acknowledge here the correction by M. Magendie, in regard to the office of the suborbital division of this nerve, since it has given occasion to the revisal of the anatomy*.

We were involved in great confusion by the discovery of new branches of nerves and of ganglions, through which we had no guide, until we formed a correct arrangement of the whole system. It is satisfactory to find that the ideas first suggested by a comparison between the roots of the nerves and their complex distribution in the face and neck are correct, when tried by a minute investigation of the internal nerves of the head; and that the conclusions drawn from the anatomy, are confirmed both by experiment and by a knowledge of the effects of injuries and of disease in the human frame.

Additional Note.—As the most important fact in this paper is that ascertained by experiments on the fifth nerve, I am bound to say by whom they were made, and for what purpose.

To my late brother-in-law Mr. John Shaw, whom I educated, I have been indebted through the whole of this inquiry. He had long been acquainted in the most intimate manner with my pursuits. He had repeated my experiments on the roots of the spinal nerves, confirming the results,—that the anterior roots when irritated caused the muscles to contract, and that the posterior roots had no such influence.

He assisted me in my experiments on the nerves of the face, which were for the purpose of establishing that the fifth pair resembled the nerves of the spine, and at the same time proving, what was incomplete from the experiments on the spinal nerves, that a ganglion on one of the roots of a nerve is no cause of

* M. Magendie says, "Le résultat que nous avons obtenu s'accorde parfaitement avec celui que nous venons de rapporter, à l'exception toutefois de l'influence de la section de sous-orbitaire sur la mastication, influence qui n'a pas été evidente pour moi."—Journal de Physiologie, 1821.

interruption to sensation, but the sign that it bestows sensibility; making certain what could be only assumed from the experiments on the spinal nerves.

But he was acquainted also with my opinions drawn from the distribution of the nerves in the body contrasted with the anatomy of their roots. And when the correctness of these opinions was established by experiment, he let no opportunity pass of advocating and supporting them. In collecting information and making dissections he was ever active, as all the real students educated with him will testify. It was in the fervour of his zeal that he went to Paris and explained the arrangement by which I distinguished the nerves, and repeated my experiments with M. Magendie and others at Charenton near Paris in 1821.

At this time an idea was thrown out that the fifth nerve was no more than the sensitive nerve of the face accidentally separated from the muscular nerve (the portio dura). Perceiving that if this notion prevailed we should be thrown back into our former state of confusion, and to put the matter beyond all question, Mr. Shaw performed those experiments which are contained in this paper,—experiments which in the gentleness of his nature he would have hesitated to make from their severity, but for their being imperatively called for.

Had Mr. Shaw lived, this subject would have been further advanced. Whilst his excellent judgement and indefatigable exertions aided me in every difficulty, his gratification in witnessing the progress of these inquiries was a reward beyond what I have now to look for.

Explanation of Plate VIII.

In this figure the superficial nerves of the face are turned off, and the distribution of the third division of the fifth to the muscles of the jaws and cheek exposed.

A. The portio dura of the seventh or respiratory nerve of the face coming out from the stylomastoid foramen; the principal branches are cut and folded forwards.

B. The trunk of the portio dura of the seventh, dissected off the face and pinned out, while it is left at its connections with the branches of the fifth on the cheek and lips.

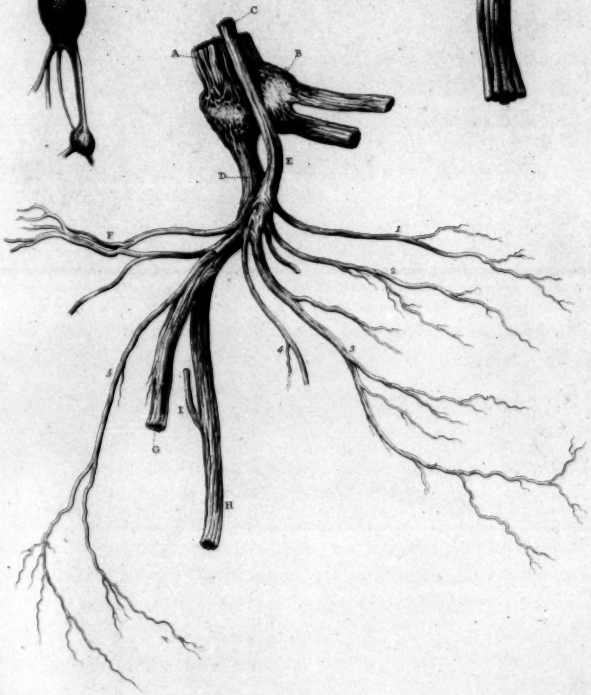
Drawn by C. Bell.

Engraved by T. Bonner.

Fig. 3.

Fig. 2.

Fig. 1.



C. The branch of the third division of the fifth nerve, which joins the plexus of the portio dura before the ear. Some experimenters, ignorant of this junction of a sensitive nerve with the muscular nerve, have occupied themselves with experiments to ascertain the degree of sensibility of the portio dura.

D. In this figure the masseter muscle is dissected from the jaw-bone and lifted up to show D, the branch of the fifth pair of nerves going into the muscle.

E. The Ramus Buccinalis-labialis, that branch of the fifth nerve which goes to the buccinator, triangularis, levator labiorum, and orbicularis muscles.

F. That branch of the fifth nerve which separating from the mandibulo-labralis goes to the muscles which depress the lower jaw.

G. The suborbitary nerve, a branch of the fifth nerve.

H. The mandibulo-labralis, a branch of the fifth nerve coming out from the bone to the muscles and integuments of the lip and chin.

I. A branch of the fifth nerve descending from the orbit.

D, E, F, are muscular branches of the fifth nerve, and are motor nerves. C, G, H, I, are sensitive branches of the same nerve which join the branches of the portio dura in its universal distribution; and although these branches of the fifth enter the muscles, they possess no power over their motions. B is the portio dura, which, though taking the same course with the last, is for a different purpose; while it is a motor nerve, by its association with the respiratory nerves, it is enabled to excite those actions of the face and lips which are necessarily connected with the act of breathing.

Explanation of Plate IX.

Fig. 1. Represents the fifth nerve dissected out and seen on its lower surface.

A. The posterior or sensitive root before it forms the ganglion.

B. The Gasserian ganglion.

C. The anterior or motor root of the nerve passing the ganglion.

D. The third or lower maxillary division of the fifth nerve.

E. The motor portion joining the lower maxillary nerve and forming a plexus with it. From this plexus go off the muscular nerves to the muscles of the jaw, viz.

1. Temporalis.

2. Massetericus.
3. Buccinalis labialis.
4. Pterygoideus.
5. Mylo-hyoideus.

F. Division which joins the portio dura.

G. Mandibulo-labralis.

H. Gustatory nerve.

I. The chorda tympani.

Fig. 2. This figure represents the ganglion on one of the spinal nerves, to show its resemblance to the ganglion of the fifth nerve in every particular.

A. The posterior or sensitive root of the nerve.

B. The ganglion formed upon the posterior root.

C. The anterior or motor root of the nerve; this arises in minute branches which join to form the larger subdivisions, whilst the posterior root is composed of simple and abrupt portions. This division joins the sensitive division beyond the ganglion exactly in the same manner that the motor portion of the fifth joins the lower maxillary nerve.

Fig. 3. Represents one of the ganglions of the sympathetic nerve to show how different it is from those on the symmetrical system of nerves. In fig. 1 and 2 the nerve on entering the ganglion and escaping from it, is separated into branches in a manner very different from the mode in which the sympathetic nerve joins or forms its ganglions*.

* Authors who have treated of the anatomy of the ganglions, have not distinguished between the two classes of ganglions as belonging to the sensitive and sympathetic systems of nerves.

XXVII. *On the reduction to a vacuum of* Captain KATER's *convertible pendulum.* *By* Captain EDWARD SABINE, *of the* Royal Artillery, *Secretary to the Royal Society.*

Read June 18, 1829.

THE convertible pendulum with which Captain KATER made his celebrated and very admirable experiments, on the length of the pendulum vibrating seconds in vacuo in Portland Place, was deposited, after the completion of those experiments, in the cabinet of the Royal Society.

The experiments of Captain KATER were made, as is well known, in the free air of the ordinary atmosphere; and the influence of the air in retarding the vibrations, and thereby interfering with the simple effect of the earth's attraction on the pendulum, was computed and allowed for on a principle universally received by mathematicians and experimentalists at that period.

It has been shown by recent investigations, theoretical and experimental, that the principle on which the reduction to a vacuum was then computed is erroneous: and it is a consequence of those investigations, that further experiments are necessary with the convertible pendulum employed by Captain KATER, in order that the true vibration in a vacuum, corresponding to the distance between its knife edges, may be known; and that the more correct length of the seconds pendulum, such as Captain KATER would himself have determined it had he been aware in 1817 of what has subsequently been discovered, may be substituted for the result published by him in the Philosophical Transactions for 1818.

The apparatus, of which an account has been presented to the Society in the present session, in which pendulums can be vibrated both in air of ordinary density and in a highly rarefied medium approaching to a vacuum, affords the means of making these further experiments. At the wish of Captain KATER, and in compliance with a request of the council of the Royal Society, I have undertaken to make them; and hope, at the commencement of the next session,

to present to the Society the result which shall be obtained, as the result of Captain KATER's pendulum.

From the principles developed in the recent investigation into the action of the air on the vibrations of a pendulum, it was to be inferred that a convertible pendulum, such as the one Captain KATER employed, would in two respects be affected by the medium in a different manner from that which he had supposed: namely, first, in respect to its presumed convertibility; for, since the amount of the retardation occasioned by the air is dependent in part on the external figure of the body vibrating, and as the two ends of the pendulum are not symmetrical, the one being furnished with a large weight, and the other with a much smaller weight and of a different form, the reduction to a vacuum will not be of the same amount when the pendulum is suspended with the great weight uppermost, as when suspended with the great weight below; and consequently the pendulum is erroneously supposed to be convertible when the vibrations in air are identical. And second, in respect to the amount of the retardation produced by the air, which would be considerably greater than the quantity computed on the simple consideration of buoyancy.

The experiments that have already been made with this pendulum in the vacuum apparatus, both in the state in which Captain KATER constructed and employed the pendulum, and with certain alterations which I have found it expedient to make in its tail pieces, have fully confirmed these inferences; and in the opinion of those whose judgement I have reason to respect, possess an interest in the elucidation, and further experimental illustration of the mode in which a medium acts on the pendulum in retarding its vibration, which makes it desirable that I should communicate the present brief account of them to the Society before the recess.

The pendulum having been conveyed to Greenwich, was examined and found in excellent order; the knife edges were as clean and apparently as perfect as when first used; the smaller weight was well secured by its screws, and the slider was at 18.6 divisions towards the greater weight: the rate of vibration on each of the knife edges was then tried by a few coincidences, and found so nearly identical, as to make it probable that little or no alteration had been made in the positions of the weight and slider since Captain KATER's experiments: and as the reduction to a vacuum for each position of the pendulum,

which was the present object of inquiry, did not require that the convertibility should be very rigorously established, the following experiments were made to determine the comparative rate, in air of ordinary density, and in highly rarefied air; first with the great weight below; and second, with the great weight uppermost.

Great weight below. April 21, 1829. Clock by DENT making 86466.00 Vibrations.

Observers.	No. of Coincid.	Standard Therm.	Barom. or Gauge.	Times of Disapp.	Times of Re-app.	Times of Coincidence.	Arc registered and true Arc.	Mean Interval.	Correction for Arc.	Reduction to 45°.	Corrected Vibrations at 45°.
				m s	m s	h m s	°				
Captain SABINE.	1	.....	.....	34 20	34 33		Div.				
	2	50.0	29.630	41 31	41 45	2 41 38.17	0.68 = 0.82				
	3	.....	.....	48 42	48 58						
	17	.....	.....	29 35	30 12			432.97	+0.36	+0.24	86067.18
Captain SABINE.	18	.....	.....	36 46	37 25						
	19	49.1	29.630	43 58	44 38	4 44 18.6	0.15 = 0.18				
	20	.....	.....	51 11	51 52						
	21	.....	.....	58 23	59 06						
		49.55	29.630; Capill. + 0.019; Reduction to 32° −0.053; = 29.596.								86067.18

April 22. Clock making 86466.00 Vibrations.

				m s	m s	h m s	°				
Mr. TAYLOR.	1	.....	.....	19 49	19 59		Div.				
	2	47.7	1.05	27 18	27 28	0 27 23	0.92 = 1.10				
	3	.....	.....	34 47	34 57			449.44	+1.60	−0.24	86082.60
Mr. TAYLOR.	13	.....	.....	49 39	49 53						
	14	47.7	1.05	57 10	57 24	1 57 16.33	0.73 = 0.88				
	15	.....	.....	04 39	04 53						
		47.7; + Index correction for a rarefied medium 0.75; = 48.45. Gauge 1.05.									86082.60

				m s	m s	h m s	°				
Mr. TAYLOR.	1	.....	.....	27 19	27 32		Div.				
	2	49.0	29.350	34 33	34 46	2 34 39.33	0.79 = 0.95				
	3	.....	.....	41 46	42 00			434.28	+0.65	−0.17	86068.26
Captain SABINE.	11	.....	.....	39 34	39 58						
	12	48.2	29.350	46 49	47 15	3 47 02.17	0.30 = 0.36				
	13	.....	.....	54 07	54 30						
		48.6	29.350; Capill. + 0.019; Reduction to 32° − 0.052; = 29.317.								86068.26

Great weight uppermost. April 23rd. Clock making 86466.00 vibrations.

Observers.	No. of Coincid.	Standard Therm.	Barom. or Gauge.	Times of Disapp.	Times of Re-app.	Times of Coincidence.	Arc registered and true Arc.	Mean Interval.	Correction for Arc.	Reduction to 45°.	Corrected Vibrations at 45°.
				m s	m s						
Captain Sabine.	1	.....	.....	55 27	55 39	h m s	Div. °	s	s	s	
	2	49.5	29.604	02 42	02 55	1 02 49.17	0.60 = 0.72	438.484	+0.59	+0.26	86072.15
	3	.....	.....	09 59	10 13						
Captain Sabine.	12	.....	.....	15 39	16 09						
	13	49.7	29.612	22 55	23 30	6 23 12.5	0.15 = 0.18				
	14	.....	.....	30 13	30 49						
		49.6	29.608; Capill. + 0.019; Réduction to 32° − 0.052; = 29.575.								86072.15

				m s	m s						
Captain Sabine.	1	.....	.....	18 22	18 31	h m s	Div. °	s	s	s	
	2	49.6	0.90	25 56	26 04	3 26 00	1.20 = 1.44				
	3	.....	.....	33 29	33 38			454.91	+1.04	+0.39	86087.27
Mr. Taylor.	34	.....	.....	28 21	28 32						
	35	48.7	3.30	35 58	36 26	7 36 12	0.25 = 0.30				
	36	.....	.....	43 33	44 02						
		49.15; + Index correction for a rarefied medium 0.75 = 49.9.								Gauge 2.10	86087.27

				m s	m s						
Mr. Taylor.	1	.....	.....	6 17	6 33	h m s	Div. °	s	s	s	
	2	49.6	29.640	13 34	13 51	8 13 42	0.98 = 1.176				
	3	.....	.....	20 51	21 06			439.1	+0.62	+0.04	86072.84
Mr. Taylor.	13	.....	.....	33 59	34 24						
	14	48.6	29.630	41 18	41 44	9 41 31.17	0.17 = 0.20				
	15	.....	.....	48 38	49 04						
		49.1	29.635; Capill. + 0.019; Reduction to 32° − 0.052; = 29.602.								86072.84

The vibrations in these experiments were as follows.

With the great weight below:

	Vibrations.	
April 21. Previous to the vibration in the rarefied medium	86067.18	Barom. 29.596
April 22. Subsequent to the vibration in the rarefied medium	86068.26	Barom. 29.317
Mean	86067.72	Barom. 29.456
The vibrations in a rarefied medium reduced to the same temperature as those in air of ordinary density	86082.60	Gauge 1.05
Difference	14.88	28.406

Whence there appears, as the result of the experiment with the great weight below, a difference of 14.88 vibrations per diem, corresponding to a difference of atmospheric pressure of 28.406 inches of mercury at 32°; the temperature of the air of full pressure being 49.07, and that of the rarefied medium 48.45.

And with the great weight uppermost:

	Vibrations.	
April 23. Previous to the vibration in the rarefied medium	86072.15	Barom. 29.575
April 23. Subsequent to the vibration in the rarefied medium	86072.84	Barom. 29.602
Mean	86072.49	Barom. 29.588
The vibrations in a rarefied medium reduced to the same temperature as those in air of ordinary density	86087.27	Gauge 2.10
Difference	14.78	27.488

Whence, with the great weight uppermost, there is found a difference of 14.78 vibrations per diem, corresponding to a difference of atmospheric pressure of 27.488 inches of mercury at 32°; the temperature of the air of full pressure being 49.35, and that of the rarefied medium 49.9.

From these results we obtain 15.7 vibrations per diem as the reduction to a vacuum for the convertible pendulum, as it was used by Captain KATER, vibrating with the great weight below, in air of 49°, under a pressure of 30 inches of mercury at 32°; and 16.1 vibrations per diem, when inverted, or with the great weight uppermost, in air of like temperature and density.

According to the formula by which Captain KATER reduced the vibrations of this pendulum in air to the supposed vibrations in a vacuum, the reduction for each position of the pendulum would be the same, and the amount for air of the named temperature and density would have a little exceeded 7 vibrations per diem.

In the account of Captain KATER's experiments (Phil. Trans. 1818, page 75) it is remarked that on a sudden and considerable change having taken place in the hygrometric state of the atmosphere, vibrations which had been previously identical in each position of the pendulum ceased to be so; an effect which he attributed to an alteration in the weight of the wooden extremities of the pen-

dulum by their loss of moisture as the weather became more dry. As we now know that the general effect of the air in retarding the vibration is more than twice as great as was then imagined, and that the wooden tail pieces, in consequence of their position, have a far greater proportionate influence on the retardation than would be simply due to the diminution which they occasion in the general specific gravity of the pendulum, we perceive increased reason to agree with Captain KATER, and to apprehend so much danger of derangement from this cause, as to make it desirable to avoid altogether the employment of a material susceptible of changes from moisture.

For this reason I determined to substitute tail pieces of brass, with such an alteration in the position or size of the smaller weight as should re-establish the equality of vibration. Previously, however, to this being done, and for the purpose of illustrating more strongly the effect of the wooden tail pieces, I had them reduced to less than half their original length, 10.6 inches being taken off from each, leaving them 6.4 inches from the extremity of the brass bar. By substituting a still smaller weight (of 1925 grains) of the same metal and form, for the smallest weight used by Captain KATER (of 3325 grains), and securing it very nearly in the same position, the pendulum was again rendered nearly convertible, and experiments were made with it in the same succession as before, of which the following are the results.

With the great weight below, there was found a difference of 11.9 vibrations per diem, corresponding to a difference of atmospheric pressure of 28.741 inches of mercury at 32°, the temperature of the air of full pressure being 53.2, and of the rarefied air 53.8.

And with the great weight uppermost a difference of 14 vibrations per diem, corresponding to a difference of 28.15 inches of mercury at 32°, the temperature of the air of full pressure being 51.4, and of the rarefied air 53.8.

Whence we obtain 12.4 vibrations per diem as the reduction to a vacuum, when the wooden tail pieces were shortened from 17 inches to 6.4 inches, and the great weight was below, in air of 53°.5, under a pressure of 30 inches of mercury at 32°; and 14.9 vibrations per diem with the great weight uppermost, in air of the same temperature and density.

The wooden tail pieces were then altogether removed, and slips of brass substituted, extending 7 inches from each extremity of the bar. The pendulum

was rendered convertible in air by the small weight used by Captain KATER, screwed to the bar at a somewhat greater distance from its knife edge than in his experiments; and coincidences being observed in the usual routine, it appeared that with the great weight below, a difference took place of 11.3 vibrations per diem, corresponding to a difference of atmospheric pressure of 28.46 inches of air at 32°, the temperature of the air of full pressure being 58°, and of the rarefied air 55°.5: and with the great weight uppermost, a difference of 12 vibrations per diem, corresponding to a difference of atmospheric pressure of 28.04 inches of air at 32°, the temperature of the air of full pressure being 60°, and of the rarefied air 60°.5.

Whence we obtain when the great weight is below, 11.8 vibrations per diem as the reduction to a vacuum, in air of 60°, under a pressure of 30 inches of mercury at 32°; and 12.8 vibrations per diem when the great weight is uppermost.

Finally, recapitulating the results obtained with the different modifications of the tail pieces, we have

	Vibrations.	Vibrations.
1. With the wooden tail pieces 17 inches in length	Great weight above 16.1;	below 15.7
2. With the wooden tail pieces 6.4 inches in length	—— 14.9;	—— 12.4
3. With the brass tail pieces 7 inches in length	—— 12.8;	—— 11.8

In comparing Nos. 1 and 2 with each other, and both with No. 3, we perceive the very great effect which the employment of so light a material as wood for the tail pieces of the pendulum produces, in increasing the difference between the vibrations in air and those in a vacuum. When slips of brass were substituted for the slips of deal employed by Captain KATER, the retardation caused by the air was diminished not less than between 3 and 4 vibrations per diem on the one knife edge, and upwards of 4 vibrations per diem on the other. When the wooden tail pieces were reduced to the same length, or nearly so, as those of brass, the retardation with them was still found greater than with the brass tail pieces of similar size and figure, by 2 vibrations per diem on the one knife edge, and 1 vibration per diem on the other. The retardations were in all cases much more considerable than would have been

computed on the simple consideration of buoyancy: they were particularly so in those instances in which the extremities of the pendulum were of the lighter material; for the increase in the retardation in those instances much exceeded the proportion due to the diminution of the general specific gravity of the pendulum, occasioned by the addition of the small portions of wood.

In viewing the comparative retardations in the two positions of the pendulum, in each of these experiments, we find a confirmation of the inference, noticed in the earlier part of this paper, that in consequence of the want of symmetry in the two ends, the reduction to a vacuum ought not to be the same in the two positions, of the pendulum. We have also a curious exemplification of the influence of the addition of equal portions of matter at each extremity of the pendulum, in diminishing the difference in the retardation occasioned by the disparity in the form and size of the weights. With the short wooden tail pieces, the difference (which with no tail pieces at all would probably have exceeded 3 vibrations per diem) amounted to 2.5 vibrations. With the brass tail pieces it was lessened to 1 vibration. And with the wooden tail pieces of their original length, the effect of the inequality of the weights was almost altogether counterbalanced, the retardation being within half a vibration the same in each position of the pendulum.

Finally, it is curious to perceive how much the result obtained by Captain KATER, as the length of the seconds pendulum, depended on the mere accidental circumstance of the addition of tail pieces to his experimental pendulum: had the circumstances of the experiment been varied in regard to the tail pieces; had they been of brass for example;—or being of wood, had they been of any other length than that which was determined by the accidental circumstance of the relative heights of the clock, and pendulum support;—or had they been altogether omitted and the coincidences observed by means of the bar itself,—a widely different result would in each of these cases have been arrived at.

XXVIII. *On the geometrical representation of the powers of quantities, whose indices involve the square roots of negative quantities. By the Rev.* JOHN WARREN, *M.A. late Fellow and Tutor of Jesus College, Cambridge. Communicated by the President.*

Read June 4, 1829.

ABOUT three months ago I wrote a paper intitled "Consideration of the objections raised against the geometrical representation of the square roots of negative quantities," which paper was communicated to the Royal Society by Dr. YOUNG, and read on the 19th of February last. At that time I had only discovered the manner of representing geometrically quantities of the form $a + b\sqrt{-1}$, and of geometrically adding and multiplying such quantities, and also of raising them to powers, either whole or fractional, positive or negative; but I was not then able to represent geometrically quantities of the form $a + b\sqrt{-1}^{m+n\sqrt{-1}}$, that is, quantities raised to powers, whose indices involve the square roots of negative quantities. My attention, however, has since been drawn to these latter quantities in consequence of an observation which I met with in M. MOUREY's work on this subject (the work which I mentioned in my former paper); the observation is as follows:

"Les limites dans lesquelles je me suis restreint m'ont forcé à passer sous silence plusieurs espèces de formules, telles sont celles-ci

$$a^{\sqrt{-1}},\ a_{\sqrt{-1}},\ \sin(\sqrt{-1})\ \&c.,\ \&c.,\ \&c.$$

Je les discute amplement dans mon grand ouvrage, et je démontre que toutes expriment des lignes directives situées sur le même plan que 1 et $\underset{1}{1}$."

where $a_{\sqrt{-1}}$ and $\underset{1}{1}$ in M. MOUREY's notation signify respectively $a\left(\frac{1}{1}\right)^{\frac{\sqrt{-1}}{4}}$ and $\left(\frac{1}{1}\right)^{\frac{1}{4}}$ according to my notation.

From this observation it was evident that M. Mourey had arrived at the geometrical representation of all algebraic quantities whatever, and that in a larger work he entered fully into the subject; but from his Preface it appeared also, that this larger work existed only in manuscript, and that circumstances would not permit the author to publish it at present. I was induced therefore to pursue my own investigations further; and the result was, that I found (as M. Mourey had stated) that all algebraic quantities whatever are capable of geometrical representation, and are represented by lines all situated in the same plane: and my view in what I am now writing is to communicate this result to algebraists.

This paper, therefore, is intended as a continuation of my "Treatise on the geometrical representation of the square roots of negative quantities;" and the object of it is to extend the geometrical representation to the powers of quantities, whose indices involve the square roots of negative quantities.

Art. 1. Def.) Mathematicians apply the words 'possible' and 'impossible' to algebraic quantities, the former signifying either positive or negative quantities; the latter, quantities involving the square roots of negative quantities. In this sense, as a matter of convenience, these words will be used in this paper; it being understood at the same time that by the word 'impossible' no impossibility is necessarily implied, but on the contrary, that the quantities called impossible have a real existence, and are capable of geometric representation.

2. Def.) Logarithms, according to the common definition given by mathematicians, must be possible quantities; therefore as a general definition of logarithms will be given in this paper, it will be desirable for the sake of distinction to give a more limited name to the common definition of logarithms, and accordingly they will be called possible logarithms; also for the like reason, the powers of quantities according to the common definition of powers, will be called possible powers.

3. Def.) Let ϱ be any quantity whatever, and let ϱ be inclined to unity at an angle $= \theta$, and let r be a positive quantity equal in length to ϱ, and let the possible hyperbolic logarithm of r be v; then $v + \theta\sqrt{-1}$ is called the general hyperbolic logarithm of ϱ, and expressed thus ϱ'.

4. Cor. 1.) $\int \frac{d\varrho}{\varrho} = \underset{\circ}{\varrho'}$

For (by Treatise, Art. 168.) $\int \frac{d\varrho}{\varrho} = v + \theta\sqrt{-1}$.

$\therefore \int \frac{d\varrho}{\varrho} = \underset{\circ}{\varrho'}$

5. Cor. 2.) If $v + \theta\sqrt{-1} = \underset{\circ}{\varrho'}$; then $v + \overline{\theta + pc}\sqrt{-1}$ is also a value of $\underset{\circ}{\varrho'}$, where p is any whole number, either positive or negative, and c is the circumference of a circle whose radius = 1.

For, since ϱ is inclined to unity at an angle $= \theta$, it is also inclined to unity at an angle $= \theta + pc$,

$\therefore$ (by Art. 3.) $v + \overline{\theta + pc} \cdot \sqrt{-1} = \underset{\circ}{\varrho'}$

6. Cor. 3.) Hence, if θ be positive and less than c, $\underset{p}{\varrho'} = v + \overline{\theta + pc} \cdot \sqrt{-1}$.

7. Cor. 4.) Hence $\underset{p}{\varrho'} = \underset{q}{\varrho'} + \overline{p - q} \cdot c\sqrt{-1}$.

For $\underset{p}{\varrho'} = v + \overline{\theta + pc} \cdot \sqrt{-1}$,

and $\underset{q}{\varrho'} = v + \overline{\theta + qc} \cdot \sqrt{-1}$,

$\therefore \underset{p}{\varrho'} = \underset{q}{\varrho'} + \overline{p - q} \cdot c\sqrt{-1}$.

8. Cor. 5.) Hence $\underset{p}{\varrho'} = \underset{\circ}{\varrho'} + pc\sqrt{-1}$.

9. Cor. 6.) If ϱ be a positive quantity, $\underset{\circ}{\varrho'}$ = possible hyperbolic logarithm of ϱ.

10. Cor. 7.) Hence, if ϱ be any quantity whatever, and r be a positive quantity equal in length to ϱ, and ϱ be inclined to unity at an angle $= \theta$, θ being positive and less than c; $\underset{p}{\varrho'} = \underset{\circ}{r'} + \overline{\theta + pc} \cdot \sqrt{-1}$.

11. Def.) Let a and ϱ be any two quantities whatever, then $\frac{\varrho'}{a'}$ is called the general logarithm of ϱ in a system whose base is a.

12. Cor. 1.) If $\underset{p}{a}$ be the base of a system of general logarithms, then the general logarithm of $\underset{p}{a}$ in that system is 1.

13. Cor. 2.) Let E be a quantity such that $\underset{\circ}{\mathrm{E}'} = 1$, and let ϱ be any quantity whatever; then ϱ' = general logarithm of ϱ in a system whose base is $\underset{\circ}{\mathrm{E}}$.

For, let v = general logarithm of ϱ in a system whose base is $\underset{\circ}{\mathrm{E}}$,

then (by Art. 11.) $v = \frac{\varrho'}{\underset{\circ}{\mathrm{E}}} = \frac{\varrho'}{1} = \varrho'$.

14. Cor. 3.) Hence the quantity $\underset{\circ}{\mathrm{E}}$ in the preceding article is the base of the system of general hyperbolic logarithms.

15. Def.) Let a and m be any quantities whatever, and let ϱ be a quantity such that one of its general logarithms in a system, whose base is a, is m; then ϱ is called the m^{th} general power of a, and expressed thus a^{m}; and the m^{th} general power of a is expressed thus $\left(\underset{p}{a}\right)^{m}$.

16. Cor.) Hence $\varrho' = m\,\underset{p}{a'}$.

17. Let a be any quantity whatever, and let b be a positive quantity in length equal to a, and let a be inclined to unity at an angle $= \alpha$, where α is positive and less than c the circumference of the circle, and let $\varrho = \left(\underset{p}{a}\right)^{m}$; then, if m be a possible quantity, ϱ will be in length $= \left(\underset{\circ}{b}\right)^{m}$, and will be inclined to unity at an angle $= m \,.\, \overline{\alpha + p\,c} \,.\, \sqrt{-1}$.

For, since $\varrho = \left(\underset{p}{a}\right)^{m}$, one of the values of ϱ' is $m\,\underset{p}{a'}$,

Let $\underset{\varphi}{\varrho'}$ be that value of ϱ',

then $\underset{\varphi}{\varrho'} = m\,\underset{p}{a'}$

$$= m\,\underset{\circ}{b'} + m \,.\, \overline{\alpha + p\,c} \,.\, \sqrt{-1};$$

Let r be a positive quantity, in length $= \varrho$, and let ϱ be inclined to unity at an angle $= \theta$, θ being positive and less than c,

then $\underset{\varphi}{\varrho'} = \underset{\circ}{r'} + \overline{\theta + q\,c} \,.\, \sqrt{-1}$,

$$\therefore \underset{\circ}{r'} + \overline{\theta + q\,c} \,.\, \sqrt{-1} = m\,\underset{\circ}{b'} + m \,.\, \overline{\alpha + p\,c} \,.\, \sqrt{-1},$$

$$\therefore \underset{\circ}{r'} = m\,\underset{\circ}{b'}, \text{ and } \theta + q\,c = m \,.\, \overline{\alpha + p\,c}$$

$$\therefore r = \left(\underset{\circ}{b}\right)^{m},$$

$\therefore \varrho$ is in length $= \left(\underset{\circ}{b}\right)^{m}$;

But ϱ is inclined to unity at an angle $= \theta + q\,c$,

$\therefore$ ϱ is inclined to unity at an angle $= m \,.\, \overline{\alpha + p\,c}$;

$\therefore$ ϱ is in length $= \left(\underset{o}{b}\right)^m$, and is inclined to unity at an angle $= m \,.\, \overline{\alpha + p\,c}$.

18. Let a be a positive quantity, and let ϱ be the m^{th} possible power of $\underset{o}{a}$; then ϱ will also be the m^{th} general power of $\underset{o}{a}$.

For since $\underset{o}{a}$ is a positive quantity, and ϱ the m^{th} possible power of $\underset{o}{a}$, (by Treatise, Art. 65.) ϱ is a positive quantity;

Also, from the nature of possible logarithms, since ϱ is the m^{th} possible power of $\underset{o}{a}$; possible hyperbolic logarithm of $\varrho = m \times$ possible hyperbolic logarithm of $\underset{o}{a}$,

that is (by Art. 9.) $\underset{o}{\varrho'} = m\,\underset{o}{a'}$,

$\therefore$ (by Art. 15.) ϱ is the m^{th} general power of $\underset{o}{a}$.

19. Let a be any quantity whatever, and let ϱ be the m^{th} possible power of $\underset{p}{a}$, then ϱ will also be the m^{th} general power of $\underset{p}{a}$.

For let b be a positive quantity, in length $= a$,

and let a be inclined to unity at an angle $= \alpha$, α being positive and less than c the circumference of the circle,

then, since ϱ is the m^{th} possible power of $\underset{p}{a}$,

(by Treatise, Art. 59, 60.) length of $\varrho = m^{\text{th}}$ possible power of b

$= $ (by Art. 18.) $\left(\underset{o}{b}\right)^m$;

And (by Treatise, Art. 63, 64.) ϱ is inclined to unity at an angle $= m \,.\, \overline{\alpha + p\,c}$;

$\therefore$ (by Art. 17.) ϱ is the m^{th} general power of $\underset{p}{a}$.

20. Let $\left(\underset{p}{a}\right)^m = \left(\underset{q}{a}\right)^m$, where a is any quantity whatever, and m either irrational or impossible; then $p = q$.

For let $\left(\underset{p}{a}\right)^m$ or $\left(\underset{q}{a}\right)^m = b$,

then, since $b = \left(\underset{p}{a}\right)^m$, one value of $\underset{p}{b'} = m\,\underset{p}{a'}$,

let $\underset{z}{b'}$ be that value,

then $\underset{x}{b'} = m\,\underset{p}{a'}$;

In like manner let $\underset{y}{b'} = m\,\underset{q}{a'}$;

then $\underset{x}{b'} - \underset{y}{b'} = m\,\underset{p}{a'} - m\,\underset{q}{a'}$,

$\therefore$ (by Art. 7.) $\overline{x - y} \cdot c\sqrt{-1} = m \cdot \overline{p - q} \cdot c\sqrt{-1}$,

$\therefore x - y = m \cdot p - q$,

where m is either irrational or impossible; therefore, since x, y, p, q are either $= 0$ or are whole numbers either positive or negative, the conditions of the equation cannot be satisfied unless $p = q$,

$\therefore p = q$.

21. Let $\left(\underset{p}{a}\right)^m = b$, and let $\left(\underset{q}{b}\right)^n = f$, where a, m, n, are any quantities whatever;

then, if $\underset{q}{b'}$ be that value of b', which is equal to $m\,\underset{p}{a'}$, $\left(\underset{p}{a}\right)^{mn} = f$.

For, since $f = \left(\underset{q}{b}\right)^n$

$f' = n\,\underset{q}{b'}$

$= m\,n\,\underset{p}{a'}$

$\therefore f = \left(\underset{p}{a}\right)^{mn}$.

22. $\left(\underset{o}{\mathrm{E}}\right)^{m+n\sqrt{-1}} = \left(\underset{o}{\mathrm{E}}\right)^m \left(\underset{1}{1}\right)^{\frac{n}{c}}$, where m and n are any possible quantities whatever, and c is the circumference of the circle, and E the base of the hyperbolic logarithms.

For let $\left(\underset{o}{\mathrm{E}}\right)^{m+n\sqrt{-1}} = \underset{\varrho}{\varrho'}$,

then one of the values of $\underset{\varrho}{\varrho'}$ is $m + n\sqrt{-1}$,

let $\underset{q}{\varrho'}$ be that value of $\underset{\varrho}{\varrho'}$,

then $\underset{q}{\varrho'} = m + n\sqrt{-1}$;

let r be a positive quantity, in length $= \varrho$, and let ϱ be inclined to unity at an angle $= \theta$, θ being positive and less than c the circumference of the circle,

then $\underset{q}{\varrho'} = \underset{o}{r'} + \overline{\theta + q\,c} \cdot \sqrt{-1}$;

$$\therefore r' + \overline{\theta + q\,c} \cdot \sqrt{-1} = m + n\sqrt{-1},$$

$$\therefore r' = m, \text{ and } \theta + q\,c = n,$$

$$\therefore r = \left(\begin{matrix}\mathrm{E}\\ \circ\end{matrix}\right)^{m};$$

$$\text{But } \varrho = r\left(\begin{matrix}1\\ 1\end{matrix}\right)^{\frac{\theta}{c}} = r\left(\begin{matrix}1\\ 1\end{matrix}\right)^{\frac{\theta+qc}{c}},$$

$$\therefore \varrho = r \cdot \left(\begin{matrix}1\\ 1\end{matrix}\right)^{\frac{n}{c}}$$

$$= \left(\begin{matrix}\mathrm{E}\\ \circ\end{matrix}\right)^{m} \cdot \left(\begin{matrix}1\\ 1\end{matrix}\right)^{\frac{n}{c}}$$

$$\therefore \left(\begin{matrix}\mathrm{E}\\ \circ\end{matrix}\right)^{m+n\sqrt{-1}} = \left(\begin{matrix}\mathrm{E}\\ \circ\end{matrix}\right)^{m} \cdot \left(\begin{matrix}1\\ 1\end{matrix}\right)^{\frac{n}{c}}.$$

23. Cor.) Hence $\left(\begin{matrix}\mathrm{E}\\ \circ\end{matrix}\right)^{n\sqrt{-1}} = \left(\begin{matrix}1\\ 1\end{matrix}\right)^{\frac{n}{c}}$.

24. $\left(\begin{matrix}\mathrm{E}\\ \circ\end{matrix}\right)^{m} \cdot \left(\begin{matrix}\mathrm{E}\\ \circ\end{matrix}\right)^{n} = \left(\begin{matrix}\mathrm{E}\\ \circ\end{matrix}\right)^{m+n}$, where m and n are any quantities whatever, and E the base of the hyperbolic logarithms.

For let $\left.\begin{matrix} m = p + q\sqrt{-1} \\ n = s + t\sqrt{-1} \end{matrix}\right\}$ where p, q, s, t are possible quantities,

then $\left(\begin{matrix}\mathrm{E}\\ \circ\end{matrix}\right)^{m} \cdot \left(\begin{matrix}\mathrm{E}\\ \circ\end{matrix}\right)^{n} = \left(\begin{matrix}\mathrm{E}\\ \circ\end{matrix}\right)^{p+q\sqrt{-1}} \cdot \left(\begin{matrix}\mathrm{E}\\ \circ\end{matrix}\right)^{s+t\sqrt{-1}}$

$$= \left(\begin{matrix}\mathrm{E}\\ \circ\end{matrix}\right)^{p} \cdot \left(\begin{matrix}1\\ 1\end{matrix}\right)^{\frac{q}{c}} \cdot \left(\begin{matrix}\mathrm{E}\\ \circ\end{matrix}\right)^{s} \cdot \left(\begin{matrix}1\\ 1\end{matrix}\right)^{\frac{t}{c}}$$

$$= \text{(by Treatise, Art. 88.)} \left(\begin{matrix}\mathrm{E}\\ \circ\end{matrix}\right)^{p+s} \cdot \left(\begin{matrix}1\\ 1\end{matrix}\right)^{\frac{q+t}{c}}$$

$$= \left(\begin{matrix}\mathrm{E}\\ \circ\end{matrix}\right)^{p+s+\overline{q+t}\cdot\sqrt{-1}}$$

$$= \left(\begin{matrix}\mathrm{E}\\ \circ\end{matrix}\right)^{m+n}.$$

25. $\left(\begin{matrix}a\\ p\end{matrix}\right)^{m} \cdot \left(\begin{matrix}a\\ p\end{matrix}\right)^{n} = \left(\begin{matrix}a\\ p\end{matrix}\right)^{m+n}$, where a, m, n are any quantities whatever.

For let $\underset{p}{a'} = s$,

then $\left(\left(\underset{p}{a}\right)^m\right)' = m\,s$,

$\therefore \left(\underset{p}{a}\right)^m = \left(\underset{\circ}{\mathrm{E}}\right)^{m\,s}$,

In like manner $\left(\underset{p}{a}\right)^n = \left(\underset{\circ}{\mathrm{E}}\right)^{n\,s}$, and $\left(\underset{p}{a}\right)^{m+n} = \left(\underset{\circ}{\mathrm{E}}\right)^{\overline{m+n}\,.\,s}$,

$$\therefore \left(\underset{p}{a}\right)^m \cdot \left(\underset{p}{a}\right)^n = \left(\underset{\circ}{\mathrm{E}}\right)^{m\,s} \cdot \left(\underset{\circ}{\mathrm{E}}\right)^{n\,s}$$

$$= \left(\underset{\circ}{\mathrm{E}}\right)^{\overline{m+n}\,.\,s}$$

$$= \left(\underset{p}{a}\right)^{m+n}.$$

26. $\dfrac{1}{\left(\underset{p}{a}\right)^m} = \left(\underset{p}{a}\right)^{-m}$, where a and m are any quantities whatever.

For let $\underset{p}{a'} = n$,

then $\left(\left(\underset{p}{a}\right)^m\right)' = m\,n$, and $\left(\left(\underset{p}{a}\right)^{-m}\right)' = -\,m\,n$,

$\therefore \left(\underset{p}{a}\right)^m = \left(\underset{\circ}{\mathrm{E}}\right)^{m\,n}$, and $\left(\underset{p}{a}\right)^{-m} = \left(\underset{\circ}{\mathrm{E}}\right)^{-m\,n}$;

Let $m\,n = s + t\sqrt{-1}$, where s and t are possible quantities,

then $\left(\underset{p}{a}\right)^m = \left(\underset{\circ}{\mathrm{E}}\right)^{s+t\sqrt{-1}} = \left(\underset{\circ}{\mathrm{E}}\right)^{s} \cdot \left(\underset{1}{1}\right)^{\frac{t}{c}}$,

$$\therefore \frac{1}{\left(\underset{p}{a}\right)^m} = \frac{1}{\left(\underset{\circ}{\mathrm{E}}\right)^{s} \cdot \left(\underset{1}{1}\right)^{\frac{t}{c}}} = \left(\underset{\circ}{\mathrm{E}}\right)^{-s} \cdot \left(\underset{1}{1}\right)^{-\frac{t}{c}} = \left(\underset{\circ}{\mathrm{E}}\right)^{-s-t\sqrt{-1}}$$

$$= \left(\underset{\circ}{\mathrm{E}}\right)^{-m\,n}$$

$$= \left(\underset{p}{a}\right)^{-m}.$$

27. Let a and b be any quantities whatever, and f a quantity such that $\underset{p}{a'} + \underset{q}{b'} = \underset{s}{f'}$; then $a\,b = f$.

For let $\underset{p}{a'} = x$, and $\underset{q}{b'} = y$,

then $\underset{s}{f'} = x + y$,

$$\therefore a = \left(\underset{\circ}{\mathrm{E}}\right)^{x}, b = \left(\underset{\circ}{\mathrm{E}}\right)^{y}, f = \left(\underset{\circ}{\mathrm{E}}\right)^{x+y},$$

$$\therefore a\,b = \left(\underset{\circ}{\mathrm{E}}\right)^{x} \cdot \left(\underset{\circ}{\mathrm{E}}\right)^{y} = \left(\underset{\circ}{\mathrm{E}}\right)^{x+y} = f.$$

28. Let a and b be any quantities whatever, and f a quantity such that $\underset{p}{a'} - \underset{q}{b'} = \underset{s}{f'}$; then $\frac{a}{b} = f$.

For, since $\underset{p}{a'} - \underset{q}{b'} = \underset{s}{f'}$,

$$\underset{p}{a'} = \underset{q}{b'} + \underset{s}{f'},$$

$\therefore$ (by preceding Art.) $a = b\,f$,

$$\therefore \frac{a}{b} = f.$$

29. Let a and b be any quantities whatever, and let a be inclined to unity at an angle $= \alpha$, and b at an angle $= \beta$, α and β being each positive and less than c the circumference of the circle, and let $a\,b = f$;

then $\underset{p}{a'} + \underset{q}{b'} = \underset{p+q}{f'}$, if $\alpha + \beta$ be less than c,

$$= \underset{p+q+1}{f'}, \text{ if } \alpha + \beta \text{ be not less than } c.$$

For let g be a positive quantity, in length $= a$,

$$h \text{ - - - - - - - - - - - - - - - - - } = b,$$

then $a = g\left(\underset{1}{1}\right)^{\frac{\alpha}{c}}$,

$$b = h\left(\underset{1}{1}\right)^{\frac{\beta}{c}},$$

$$\therefore f = g\,h\left(\underset{1}{1}\right)^{\frac{\alpha+\beta}{c}};$$

also $\underset{p}{a'} = \underset{\circ}{g'} + \overline{\alpha + p\,c} \cdot \sqrt{-1}$,

$$\underset{q}{b'} = \underset{\circ}{h'} + \overline{\beta + q\,c} \cdot \sqrt{-1},$$

$$\therefore \underset{p}{a'} + \underset{q}{b'} = \underset{\circ}{g'} + \underset{\circ}{h'} + \overline{\alpha + \beta + p + q} \cdot c \cdot \sqrt{-1};$$

but $\underset{\circ}{g'}$ = possible hyperbolic logarithm of g,

$\underset{\circ}{h'}$ = - - - - - - - - - - - - - - - - - - h,

$\therefore \underset{\circ}{g'} + \underset{\circ}{h'}$ = - - - - - - - - - - - - - - - - - - $g\,h$

$= \left(\underset{\circ}{g\,h}\right)'$;

$\therefore \underset{p}{a'} + \underset{q}{b'} = \underset{\circ}{g\,h'} + \overline{\alpha + \beta + \overline{p+q}\,.\,c}\,.\sqrt{-1}$;

but, since $g\,h\,.\left(\frac{1}{1}\right)^{\frac{\alpha+\beta}{c}} = f$,

$\left(\underset{\circ}{g\,h}\right)' + \overline{\alpha+\beta}\,\sqrt{-1} = \underset{\circ}{f'}$, if $\alpha + \beta$ be less than c,

$= \underset{1}{f'}$, - - - - - - not less - - ,

$\therefore \left(\underset{\circ}{g\,h}\right)' + \overline{\alpha + \beta + \overline{p+q}\,.\,c}\,.\sqrt{-1} = \underset{p+q}{f'}$, if $\alpha + \beta$ be less than c,

$= \underset{p+q+1}{f'}$, - - - - - not less - -;

$\therefore \underset{p}{a'} + \underset{q}{b'} = \underset{p+q}{f'}$, if $\alpha + \beta$ be less than c,

$= \underset{p+q+1}{f'}$, - - - - - not less - - .

30. Cor.) Hence if a and b be any quantities whatever, and a be inclined to unity at an angle $= \alpha$, and b at an angle $= \beta$, α and β being each positive and less than c the circumference of the circle, and $\frac{a}{b} = f$;

then $\underset{p}{a'} - \underset{q}{b'} = \underset{p-q}{f'}$, if α be not less than β,

$= \underset{p-q-1}{f'}$, if α be less than β.

31. Let a and b be any quantities whatever, and let a be inclined to unity at an angle $= \alpha$ and b, at an angle $= \beta$, α and β being each positive and less than c the circumference of the circle, and let $a\,b = f$, and let m be any quantity whatever;

then $\left(\underset{p}{a}\right)^m . \left(\underset{q}{b}\right)^m = \left(\underset{p+q}{f}\right)^m$, if $\alpha + \beta$ be less than c

$= \left(\underset{p+q+1}{f}\right)^m$, - - - - not less - - .

For, first, let $\alpha + \beta$ be less than c,

then (by Art. 29.) $\underset{p}{a'} + \underset{q}{b'} = \underset{p+q}{f'}$,

$$\therefore m \underset{p}{a'} + m \underset{q}{b'} = m \underset{p+q}{f'},$$

$$\therefore \left(\left(\tfrac{a}{p}\right)^m\right)' + \left(\left(\tfrac{b}{q}\right)^m\right)' = \left(\left(\tfrac{f}{p+q}\right)^m\right)',$$

$\therefore$ (by Art. 27.) $\left(\tfrac{a}{p}\right)^m \cdot \left(\tfrac{b}{q}\right)^m = \left(\tfrac{f}{p+q}\right)^m$;

Next, let $\alpha + \beta$ be not less than c

then (by Art. 29.) $\underset{p}{a'} + \underset{q}{b'} = \underset{p+q+1}{f'}$

$$\therefore \left(\tfrac{a}{p}\right)^m \cdot \left(\tfrac{b}{q}\right)^m = \left(\tfrac{f}{p+q+1}\right)^m.$$

32. Let a and b be any quantities whatever, and let a be inclined to unity at an angle $= \alpha$, and b at an angle $= \beta$, α and β being each positive and less than c the circumference of the circle, and let $\frac{a}{b} = f$, and let m be any quantity whatever;

then $\dfrac{\left(\tfrac{a}{p}\right)^m}{\left(\tfrac{b}{q}\right)^m} = \left(\tfrac{f}{p-q}\right)^m$, if α be not less than β,

$= \left(\tfrac{f}{p-q-1}\right)^m$, if α be less than β.

For this may be proved nearly in the same manner as the preceding article.

33. Let m be any quantity whatever, and E the base of the hyperbolic logarithms;

then $\left(\tfrac{E}{o}\right)^m = 1 + m + \frac{m^2}{1.2} + \frac{m^3}{1.2.3} + \&c.$

For let $m = p + q\sqrt{-1}$, where p and q are possible quantities,

then $\left(\tfrac{E}{o}\right)^m = \left(\tfrac{E}{o}\right)^{p+q\sqrt{-1}} = \left(\tfrac{E}{o}\right)^p \cdot \left(\tfrac{1}{1}\right)^{\frac{q}{c}}$

$$= \left(1 + p + \tfrac{p^2}{1.2} + \&c.\right) \cdot \left(1 + q\sqrt{-1} - \tfrac{q^2}{1.2} - \&c.\right)$$

$$= 1 + (p + q\sqrt{-1}) + \tfrac{(p+q\sqrt{-1})^2}{1.2} + \&c.$$

$$= 1 + m + \tfrac{m^2}{1.2} + \&c.$$

34. Let a be a quantity inclined to unity at an angle less than $\frac{c}{4}$, where c is the circumference of the circle;

then $\underset{o}{a'} = 2\left\{\frac{a-1}{a+1} + \frac{1}{3}\left(\frac{a-1}{a+1}\right)^3 + \frac{1}{5}\left(\frac{a-1}{a+1}\right)^5 + \&c.\right\}$.

For let $a = b\left(\frac{1}{1}\right)^{\frac{\alpha}{c}}$, where b is a positive quantity, and $\frac{\alpha}{c}$ positive and less than $\frac{1}{4}$,

then $\underset{o}{a'} = \underset{o}{b'} + \alpha\sqrt{-1}$;

Now, since $a = b\left(\frac{1}{1}\right)^{\frac{\alpha}{c}}$, we have (by Treatise, Art. 135.)

$$\left(\underset{o}{a}\right)^x = 1 + (\mathrm{B} + \alpha\sqrt{-1})\,x + \frac{(\mathrm{B} + \alpha\sqrt{-1})^2 x^2}{1.2} + \&c.$$

where x is any possible quantity,

and $\mathrm{B} = 2\left\{\frac{b-1}{b+1} + \frac{1}{3}\left(\frac{b-1}{b+1}\right)^3 + \frac{1}{5}\left(\frac{b-1}{b+1}\right)^5 + \&c.\right\}$

$$= \text{possible hyperbolic logarithm of } b$$
$$= \underset{o}{b'};$$

Also (by Treatise, Art. 132.)

$$\left(\underset{o}{a}\right)^x = 1 + \mathrm{A}\,x + \frac{\mathrm{A}^2 x^2}{1.2} + \&c.,$$

where $\mathrm{A} = 2\left\{\frac{a-1}{a+1} + \frac{1}{3}\left(\frac{a-1}{a+1}\right)^3 + \frac{1}{5}\left(\frac{a-1}{a+1}\right)^5 + \&c.\right\}$,

$\therefore$ equating the coefficients,

$$\mathrm{A} = \mathrm{B} + \alpha\sqrt{-1}$$
$$= \underset{o}{b'} + \alpha\sqrt{-1}$$
$$= \underset{o}{a'},$$

$$\therefore \underset{o}{a'} = 2\left\{\frac{a-1}{a+1} + \frac{1}{3}\left(\frac{a-1}{a+1}\right)^3 + \frac{1}{5}\left(\frac{a-1}{a+1}\right)^5 + \&c.\right\}.$$

35. Let a be a quantity inclined to unity at an angle less than $\frac{c}{4}$, and let $a - 1$ be in length less than unity;

Then $\underset{o}{a'} = a - 1 - \frac{1}{2}(a-1)^2 + \frac{1}{3}(a-1)^3 - \&c.$

For this may be proved (by Treatise, Art. 128.) nearly in the same manner as the preceding article.

36. Let a be a quantity inclined to unity at an angle greater than $\frac{3c}{4}$ and less than c;

Then $\underset{-1}{a'} = 2\left\{\frac{a-1}{a+1} + \frac{1}{3}\left(\frac{a-1}{a+1}\right)^3 + \frac{1}{5}\left(\frac{a-1}{a+1}\right)^5 + \&c.\right\}$.

For this may be proved nearly in the same manner as Art. 34.

37. Let a be a quantity inclined to unity at an angle greater than $\frac{3c}{4}$ and less than c, and let $a - 1$ be in length less than unity;

Then $\underset{-1}{a'} = a - 1 - \frac{1}{2}(a-1)^2 + \frac{1}{3}(a-1)^3 - \&c.$

For this may be proved nearly in the same manner as Art. 34.

38. Let a be a quantity inclined to unity at an angle less than $\frac{c}{4}$, and let m be any quantity whatever;

Then $\binom{a}{p}^m = 1 + (A + pc\sqrt{-1})\,m + \left(\frac{A + pc\sqrt{-1}}{1.2}\right)^2 m^2 + \&c.$

where $A = 2\left\{\frac{a-1}{a+1} + \frac{1}{3}\left(\frac{a-1}{a+1}\right)^3 + \&c.\right\}$,

or $a - 1 - \frac{1}{2}(a-1)^2 + \&c.$, if $a - 1$ be in length less than unity.

For $\underset{o}{a'} = A$

$$\therefore \underset{p}{a'} = A + pc\sqrt{-1}$$

$$\therefore \left(\binom{a}{p}^m\right)' = (A + pc\sqrt{-1})\,m,$$

$$\therefore \binom{a}{p}^m = \binom{E}{o}^{(A+pc\sqrt{-1})m}$$

$$= \text{(by Art. 33.)}\ 1 + (A + pc\sqrt{-1})\,m + \frac{(A+pc\sqrt{-1})^2}{1.2} m^2 + \&c.$$

39. Let a be a quantity inclined to unity at an angle greater than $\frac{3c}{4}$ and less than c, and let m be any quantity whatever;

Then $\binom{a}{p}^m = 1 + (A + \overline{p+1}\,c\sqrt{-1})\,m + \frac{(A + \overline{p+1}\,c\sqrt{-1})^2}{1.2} m^2 + \&c.,$

where $A = 2\left\{\frac{a-1}{a+1} + \frac{1}{3}\left(\frac{a-1}{a+1}\right)^3 + \&c.\right\}$,

or $a - 1 - \frac{1}{2}(a-1)^2 + \&c.$, if $a - 1$ be in length less than unity.

For this may be proved nearly in the same manner as the preceding article.

40. Let $u = \left(\frac{x}{p}\right)^m$, where x and m are any quantities whatever, and let m remain constant whilst x and u vary,

Then $\frac{du}{dx} = m\left(\frac{x}{p}\right)^{m-1}$.

For, since $u = \left(\frac{x}{p}\right)^m$,

$$u' = m\,x'$$

$$\therefore \text{(by Art. 4.)}\ \frac{1}{u}\cdot\frac{du}{dx} = m\cdot\frac{1}{x},$$

$$\therefore \frac{du}{dx} = m\cdot\frac{u}{x}$$

$$= m\,\frac{\left(\frac{x}{p}\right)^m}{x}$$

$$= m\left(\frac{x}{p}\right)^{m-1}.$$

41. Let $z = 1 + x$, and let z be inclined to unity at an angle $= \theta$, θ being positive and less than c, and let x be in length less than unity, and let m be any quantity whatever;

then $\left(\frac{z}{p}\right)^m = \left(\frac{1}{p}\right)^m \cdot \left\{1 + m\,x + \frac{m\cdot\overline{m-1}}{1.2}x^2 + \&c.\right\}$, if θ be less than $\frac{c}{4}$,

$$= \left(\frac{1}{p+1}\right)^m \cdot \left\{1 + m\,x + \frac{m\cdot\overline{m-1}}{1.2}x^2 + \&c.\right\}, \text{if } \theta \text{ be greater than } \frac{3c}{4}.$$

For, first, let θ be less than $\frac{c}{4}$,

and let $\left(\frac{z}{p}\right)^m = A + B\,x + C\,x^2 + \&c.$,

then differentiating

$$m\left(\frac{z}{p}\right)^{m-1} = B + 2\,C\,x + \&c.,$$

$$m\cdot\overline{m-1}\left(\frac{z}{p}\right)^{m-2} = 2\,C + \&c.,$$

$$\&c. \qquad = \&c.;$$

now let $x = 0$,

then $\left(\frac{z}{p}\right)^m$ becomes $\left(\frac{1}{p}\right)^m$

$\left(\frac{z}{p}\right)^{m-1}$ becomes $\left(\frac{1}{p}\right)^{m-1} = \left(\frac{1}{p}\right)^m$

&c. becomes &c.

$\therefore A = \left(\frac{1}{p}\right)^m$

$B = m \cdot \left(\frac{1}{p}\right)^m$

$C = \frac{m \cdot m-1}{1.2} \cdot \left(\frac{1}{p}\right)^m$

&c. = &c.

$\therefore \left(\frac{z}{p}\right)^m = \left(\frac{1}{p}\right)^m \cdot \left\{1 + m x + \frac{m \cdot m-1}{1.2} x^2 + \text{&c.}\right\}$;

Next, let θ be greater than $\frac{3c}{4}$,

In this case, when $x = 0$,

$\left(\frac{z}{p}\right)^m$ becomes $\left(\frac{1}{p+1}\right)$;

$\therefore \left(\frac{z}{p}\right)^m = \left(\frac{1}{p+1}\right)^m \cdot \left\{1 + m x + \frac{m \cdot m-1}{1.2} x^2 + \text{&c.}\right\}$.

42. Let $\left(\frac{E}{o}\right)^{x\left(\frac{E}{o}\right)^{m\sqrt{-1}}} = \varrho$, where x is a positive quantity, and m a possible quantity, and let x and ϱ vary whilst m remains constant; then ϱ will trace out a logarithmic spiral, which cuts its radii vectors at an angle $= m$.

For let r be a positive quantity in length $= \varrho$,

then, since $\left(\frac{E}{o}\right)^{m\sqrt{-1}} = \left(\frac{1}{1}\right)^{\frac{m}{c}} = \cos m + \sin m \cdot \sqrt{-1}$,

$\varrho = \left(\frac{E}{o}\right)^{x \cos m + x \sin m \cdot \sqrt{-1}}$

$= \left(\frac{E}{o}\right)^{x \cos m} \cdot \left(\frac{1}{1}\right)^{\frac{x \sin m}{c}}$,

$\therefore r = \left(\frac{E}{o}\right)^{x \cos m}$, and ϱ is inclined to unity at an angle $= x \cdot \sin m$,

$\therefore r' = x \cdot \cos m$,

$\therefore\ x = \frac{1}{\cos m} \cdot \underset{o}{r'}$,

$\therefore\ \varrho$ is inclined to unity at an angle $= \frac{\sin m}{\cos m} \cdot \underset{o}{r'} = \tan m \cdot \underset{o}{r'}$,

which is the property of a logarithmic spiral which cuts its radii vectors at an angle $= m$,

$\therefore$ the curve traced out by ϱ is a logarithmic spiral which cuts its radii vectors at an angle $= m$.

43. Cor. 1.) The logarithmic spiral in the last article will cut the positive direction at a distance $= 1$.

For let $x = 0$, then $\varrho = \left(\underset{o}{\mathrm{E}}\right)^{\circ} = 1$,

$\therefore$ one of the values of ϱ is 1,

$\therefore$ the spiral cuts the positive direction at a distance $= 1$.

44. Cor. 2.) When m is such that $\tan m = 0$, the spiral becomes a straight line; and when m is such that $\tan m$ is infinite, the spiral becomes a circle.

45. Let a be any quantity whatever, and x any possible quantity, and let $\left(\underset{p}{a}\right)^{x} = \varrho$, and let ϱ and x vary while a and p remain constant; then ϱ will trace out a logarithmic spiral.

For let $\underset{p}{a'} = n \left(\underset{o}{\mathrm{E}}\right)^{m\sqrt{-1}}$, where n is positive and m possible,

then $\left(\left(\underset{p}{a}\right)^{x}\right)' = n\, x \left(\underset{o}{\mathrm{E}}\right)^{m\sqrt{-1}}$,

$\therefore \left(\underset{o}{\mathrm{E}}\right)^{nx\left(\underset{o}{\mathrm{E}}\right)^{m\sqrt{-1}}} = \left(\underset{p}{a}\right)^{x} = \varrho$,

$\therefore$ (By Art. 42.) since m is constant, and $n\,x$ and ϱ variable, ϱ will trace out a logarithmic spiral.

46. Cor. 1.) The spiral will cut the positive direction at a distance $= 1$, and will cut its radii at an angle $= m$.

47. Cor. 2.) ϱ becomes equal to a in its $p + 1^{\text{th}}$ revolution in the spiral, reckoning from the time at which it was equal to 1.

For let b be a positive quantity in length $= a$, and let a be inclined to unity at an angle $= \alpha$, where α is positive and less than c,

then $\underset{o}{b'} + \overline{\alpha + p\,c} \cdot \sqrt{-1} = \underset{p}{a'}$

$$= n \left(\underset{o}{\mathrm{E}}\right)^{m\sqrt{-1}},$$

$$\therefore x \underset{o}{b'} + x \,.\, \overline{\alpha + p\,c} \,.\, \sqrt{-1} = n\,x \left(\underset{o}{\mathrm{E}}\right)^{m\sqrt{-1}} = n\,x \cos m + n\,x \sin m \,.\, \sqrt{-1},$$

$$\therefore x \,.\, \overline{\alpha + p\,c} = n\,x \sin m,$$

but $n\,x \sin m$ is the angle at which ϱ (considered as the radius vector of the spiral) is inclined to unity.

$\therefore x \,.\, \overline{\alpha + p\,c}$ is the angle at which ϱ, as radius vector of the spiral, is inclined to unity;

but when $x = 1$, $\varrho = a$ and angle $x \,.\, \alpha + p\,c$ becomes $\alpha + p\,c$,

and α is less than c,

$\therefore \varrho$ becomes equal to a in its $p + 1^{\text{th}}$ revolution.

48. Cor. 3.) If a be positive but not $= 1$, and $p = 0$, the spiral becomes a straight line; if $a = 1$, and p be not $= 0$, the spiral becomes a circle; and if $a = 1$, and $p = 0$, the spiral becomes a point.

49. Cor. 4.) If a be any quantity whatever, and $\underset{p}{a'} = n \left(\underset{o}{\mathrm{E}}\right)^{m\sqrt{-1}}$, n being positive and m possible; and if a logarithmic spiral be described having its pole in the origin of a, and cutting the positive direction at a distance $= 1$ and passing through the extremity of a in its $p + 1^{\text{th}}$ revolution; then the spiral will cut its radii at an angle $= m$.

50. Let $\left(\underset{p}{a}\right)^{n} \left(\underset{o}{\mathrm{E}}\right)^{m\sqrt{-1}} = \varrho$, where a is any quantity whatever, and m any possible quantity, and n any positive quantity; and let a logarithmic spiral be described having its pole in the origin of a and ϱ, and cutting the positive direction at a distance $= 1$, and passing through the extremity of a in its $p + 1^{\text{th}}$ revolution; and let a second logarithmic spiral be described having the same pole with the first spiral, and also cutting the positive direction at a distance $= 1$, and cutting the first spiral at an angle $= m$; then ϱ will be a radius vector of the second spiral.

For let $\underset{p}{a'} = l \left(\underset{o}{\mathrm{E}}\right)^{k\sqrt{-1}}$, where l is positive and k possible,

then (by Art. 49.) the first spiral will cut its radii at an angle $= k$,

and since the second spiral cuts the first at an angle $= m$,

the second spiral will cut its radii at an angle $= k + m$;

But $a' = l \left(\mathrm{E}\right)^{k\sqrt{-1}}$,

$$\therefore \left(\left(\frac{a}{p}\right)^{n}\left(\mathrm{E}\right)^{m\sqrt{-1}}\right)' = n\left(\mathrm{E}\right)^{m\sqrt{-1}} \cdot l\left(\mathrm{E}\right)^{k\sqrt{-1}} = l\,n\left(\mathrm{E}\right)^{k+m\cdot\sqrt{-1}},$$

$$\therefore \varrho = \left(\mathrm{E}\right)^{l\,n\left(\mathrm{E}\right)^{k+m\cdot\sqrt{-1}}},$$

$\therefore$ (by Art. 42 and 43.) ϱ is a radius vector of a logarithmic spiral which cuts its radii at an angle $= k + m$, and cuts the positive direction at a distance $= 1$, that is, ϱ is a radius vector of the second spiral.

51. Cor. 1.) If $\left(\frac{a}{p}\right)^{m\sqrt{-1}} = \varrho$, and m be a possible quantity; ϱ will be a radius vector in a spiral (described as in the preceding article) which cuts the spiral, in which a is, at a right angle.

52. Cor. 2.) Hence if a be a positive quantity, and $p = 0$, the spiral, in which a is, will become a straight line, and the spiral, in which ϱ is, will be perpendicular to it, that is, will be a circle; but if $a = 1$, and p be not $= 0$, the spiral, in which a is, will become a circle, and the spiral, in which ϱ is, will be perpendicular to it, that is, will be a straight line, and ϱ will be a positive quantity.

53. Let $a^{m} = \varrho$, and let a and m be any quantities whatever; then the values of ϱ are in geometric progression.

For $\left(\frac{a}{p}\right)^{m}$ represents any one value of ϱ,

$\therefore$ if we substitute for p successively 0, 1, 2, 3, &c., also -1, -2, -3, &c., we shall obtain all the values of ϱ;

Let $a' = n$,

then (by Art. 8.) $a' = n + p\,c\sqrt{-1}$,

$$\therefore \left(\left(\frac{a}{p}\right)^{m}\right)' = n\,m + p\,c\,m\sqrt{-1},$$

$$\therefore \left(\frac{a}{p}\right)^{m} = \left(\mathrm{E}\right)^{n\,m+p\,c\,m\sqrt{-1}},$$

now, if we substitute for p successively 0, 1, 2, &c., also -1, -2, &c., the values of $\left(\underset{o}{\mathrm{E}}\right)^{m+pc\sqrt{-1}}$ will be in geometric progression,

$\therefore$ the values of ϱ are in geometric progression.

54. Cor. 1.) Hence all the values of ϱ are radii vectors of the same logarithmic spiral.

55. Cor. 2.) If m be impossible or irrational, ϱ will have an infinite number of different values; but if m be rational the values will recur, and the number of different values will be equal to the denominator of m, when m is expressed as a fraction in its lowest terms.

56. Any geometric series being given, it is required to find quantities a and m, such that, a^m may have values equal to each of the terms of the series.

Let b be any term of the series, and r the common ratio,

$$\text{and let } b = \left(\underset{p}{a}\right)^m,$$

$$\text{then } b\,r = \left(\underset{p+1}{a}\right)^m = \left(\underset{p}{a}\right)^m \cdot \left(\underset{1}{1}\right)^m,$$

$$\therefore r = \left(\underset{1}{1}\right)^m;$$

Now $1' = c\sqrt{-1}$, where c is the circumference of the circle,

$$\therefore \left(\left(\underset{1}{1}\right)^m\right)' = m\,c\sqrt{-1},$$

$$\therefore r' = m\,c\sqrt{-1},$$

$$\therefore m = \frac{r'}{c\sqrt{-1}}, \therefore m \text{ is known};$$

$$\text{Now } b = \left(\underset{p}{a}\right)^m,$$

$$\therefore b' = m\,\underset{p}{a'} = \frac{r'}{c\sqrt{-1}} \cdot \underset{p}{a'},$$

$$\therefore \underset{p}{a'} = c\sqrt{-1} \cdot \frac{b'}{r},$$

$$\therefore a = \left(\underset{o}{\mathrm{E}}\right)^{c\sqrt{-1}\cdot\frac{b'}{r}}, \therefore a \text{ is known.}$$

57. Cor.) If the series be of the form $1, r, r^2$, &c., we may take $b = 1$ and $b' = 0$; then we shall have $a = 1$, and $a^m = 1^{\frac{r'}{c\sqrt{-1}}}$.

58. Ex.) Let the series be 1, 2, 4, &c.,

then $r = 2$,

and $1^{\frac{r'}{c\sqrt{-1}}} = 1^{\frac{g'}{c\sqrt{-1}}}$,

$\therefore$ the values of $1^{\frac{g'}{c\sqrt{-1}}}$ are 1, 2, 4, &c., also $\frac{1}{2}$, $\frac{1}{4}$ &c.

59. Let it be required to find the values of $\sqrt{-1}^{\sqrt{-1}}$.

$$\underset{1}{1}' = c\sqrt{-1},$$

$$\therefore \left(\underset{1}{1}^{\frac{1}{4}}\right)' = \frac{c}{4}\sqrt{-1},$$

but $\sqrt{-1} = \underset{1}{1}^{\frac{1}{4}}$,

$$\therefore \left(\sqrt{-1}\right)' = \frac{c}{4}\sqrt{-1},$$

$\therefore$ since $\frac{c}{4}$ is less than c, $\left(\underset{\circ}{\sqrt{-1}}\right)' = \frac{c}{4}\sqrt{-1}$,

$$\therefore \left(\underset{p}{\sqrt{-1}}\right)' = \frac{c}{4}\sqrt{-1} + pc\sqrt{-1} = (\tfrac{1}{4} + p)\, c\sqrt{-1},$$

$$\therefore \left(\left(\underset{p}{\sqrt{-1}}\right)^{\sqrt{-1}}\right)' = (\tfrac{1}{4} + p)\, c\sqrt{-1} \cdot \sqrt{-1} = -(\tfrac{1}{4} + p)\, c,$$

$$\therefore \left(\underset{p}{\sqrt{-1}}\right)^{\sqrt{-1}} = \left(\underset{\circ}{\mathrm{E}}\right)^{-(\frac{1}{4}+p)c} = \frac{1}{\left(\underset{\circ}{\mathrm{E}}\right)^{(\frac{1}{4}+p)c}},$$

let 0, 1, 2, &c., also -1, -2, &c., be successively substituted for p, then we have the values of $\sqrt{-1}^{\sqrt{-1}}$ as follows, viz.

$$\frac{1}{\left(\underset{\circ}{\mathrm{E}}\right)^{\frac{c}{4}}}, \frac{1}{\left(\underset{\circ}{\mathrm{E}}\right)^{\frac{5c}{4}}}, \frac{1}{\left(\underset{\circ}{\mathrm{E}}\right)^{\frac{9c}{4}}}, \text{ &c.,}$$

$$\left(\underset{\circ}{\mathrm{E}}\right)^{\frac{3c}{4}}, \left(\underset{\circ}{\mathrm{E}}\right)^{\frac{7c}{4}}, \text{ &c.,}$$

all in geometric progression.

60. From what has been demonstrated it will be manifest that all algebraic quantities may be geometrically represented, both in length and direction, by lines drawn in a given plane from a given point.

61. With respect to quantities such as $\sin(a+b\sqrt{-1})$, $\cos(a+b\sqrt{-1})$, &c., quantities strictly speaking not algebraic, it may be observed, that if it be found useful to introduce these quantities into algebra, they may without impropriety be introduced, by giving to sines, cosines, &c., algebraic definitions; thus sin A may be defined to signify

$$\frac{\left(\begin{smallmatrix}1\\1\end{smallmatrix}\right)^{\frac{A}{c}}-\left(\begin{smallmatrix}1\\1\end{smallmatrix}\right)^{-\frac{A}{c}}}{2\sqrt{-1}} \text{ or } \frac{\left(\begin{smallmatrix}E\\o\end{smallmatrix}\right)^{A\sqrt{-1}}-\left(\begin{smallmatrix}E\\o\end{smallmatrix}\right)^{-A\sqrt{-1}}}{2\sqrt{-1}}, \text{ and we shall have}$$

$$\sin(a+b\sqrt{-1})=\frac{\left(\begin{smallmatrix}E\\o\end{smallmatrix}\right)^{a\sqrt{-1}-b}-\left(\begin{smallmatrix}E\\o\end{smallmatrix}\right)^{-a\sqrt{-1}+b}}{2\sqrt{-1}}=\frac{\left(\begin{smallmatrix}E\\o\end{smallmatrix}\right)^{-b}\cdot\left(\begin{smallmatrix}1\\1\end{smallmatrix}\right)^{\frac{a}{c}}-\left(\begin{smallmatrix}E\\o\end{smallmatrix}\right)^{b}\cdot\left(\begin{smallmatrix}1\\1\end{smallmatrix}\right)^{-\frac{a}{c}}}{2\sqrt{-1}},$$

therefore, considering sines, cosines, &c., merely as algebraic quantities, we may make use of the expressions $\sin(a+b\sqrt{-1})$, $\cos(a+b\sqrt{-1})$, &c., if such expressions be found convenient in algebraic operations.

Jesus College, Cambridge,
April 22, 1829.

J. W.

61. With respect to quantities such as $\sin(a+b\sqrt{-1})$, $\cos(a+b\sqrt{-1})$, &c., quantities strictly speaking not algebraic, it may be observed, that it is found useful to introduce these quantities into algebra, they may without impropriety be introduced, by giving to sines, cosines, &c., algebraic definitions: thus $\sin A$ may be defined to signify

$$\frac{\left(\frac{1}{\cdot}\right)^{\frac{A}{\cdot}\sqrt{-1}} - \left(\frac{1}{\cdot}\right)^{-\frac{A}{\cdot}\sqrt{-1}}}{2\sqrt{-1}} \quad \text{or} \quad \frac{\left(\frac{E}{\cdot}\right)^{A\sqrt{-1}} - \left(\frac{E}{\cdot}\right)^{-A\sqrt{-1}}}{2\sqrt{-1}}, \text{ and we shall have}$$

$$\sin(a+b\sqrt{-1}) = \frac{\left(\frac{E}{\cdot}\right)^{a\sqrt{-1}-b} - \left(\frac{E}{\cdot}\right)^{-a\sqrt{-1}+b}}{2\sqrt{-1}} = \frac{\left(\frac{E}{\cdot}\right)^{-b}\left(\frac{1}{\cdot}\right)^{\frac{a}{\cdot}} - \left(\frac{E}{\cdot}\right)^{b}\left(\frac{1}{\cdot}\right)^{\frac{a}{\cdot}}}{2\sqrt{-1}},$$

therefore, considering sines, cosines, &c., merely as algebraic quantities, we may make use of the expressions $\sin(a+b\sqrt{-1})$, $\cos(a+b\sqrt{-1})$, &c., if such expressions be found convenient in algebraic operations.

Trin. College, Camb. Ibp.
April 23, 1839.

J. W.

XXIX. *An experimental examination of the electric and chemical theories of galvanism.* By WILLIAM RITCHIE, *A.M. F.R.S., Rector of the Royal Academy at Tain.*

Read May 7, 1829.

1. THE continental philosophers still continue to adopt the electric theory of galvanism proposed by VOLTA, whilst those in Britain as uniformly follow some modification of the chemical theory proposed by Dr. WOLLASTON. From this diversity of opinion we may safely conclude, that the experimental proofs for the truth of either theory are not sufficiently powerful, to command the assent of all capable of appreciating the weight of such evidence. I have therefore ventured to lay before the Society the following experiments and observations; as they appear to me to establish the truth of some modification of the chemical theory, and to demonstrate the fallacy of the principles on which the electric theory rests.

2. The fundamental principle assumed by VOLTA, and supported by his followers, is, that if dissimilar metals be brought into contact they are instantly thrown into opposite electric states. This he conceives to be a new law of nature, and claims to himself the honour of the discovery. He conceives that its truth is proved by the following experiment.

Let Z be a plate of zinc, and C a plate of copper, soldered together at the line of contact A B. Hold the plate of zinc in the hand, and touch the under plate of a delicate electric condenser (le condensateur à lames d'or) with the copper plate, whilst a moistened finger is applied to the upper plate of the instrument. Remove the compound plate and the moistened finger, and then lift the upper plate of the instrument by its insulating handle, and the slips of gold leaf will be found to diverge. Taking for granted the truth of the experiment, the conclusion which VOLTA deduced from it by no means follows as a legitimate inference. Dr. WOLLASTON has shown that a galvanic effect is produced by dissimilar metals with the moist

air of the atmosphere acting as a chemical agent and an imperfect conductor. The same fact is proved by the electric column of DeLuc. The plate of zinc becomes partially oxidized by the oxygen of the atmosphere, electricity is generated or set at liberty, and the film of moist air in contact with the two metals acts as the fluid conductor in an ordinary voltaic arrangement. If the compound plate be coated with electric cement to exclude the chemical action of the air on the zinc, I will venture to predict that no decided electric effect will take place. Until the supporters of the electric theory show by direct experiment that electric effect does take place with this modification of the apparatus, we must view the whole of their reasoning as founded on a gratuitous supposition. Having thus shown that Volta and his followers have overlooked what appears to me to be the very cause of the disturbance of electric equilibrium in the two metals, I shall now demonstrate that the other principle on which the theory is built is equally unfounded. This will appear obvious from the two following experiments.

Experiment I.

Having poured into a watch glass a quantity of diluted sulphuric acid, I placed on the surface of the fluid a piece of gold leaf, which was connected with one of the cups of a delicate galvanometer. I then placed a disc of platina foil in the fluid below the gold leaf, and connected it with the other cup of the instrument; scarcely any electro-magnetic effect was produced. Having removed the acid, I substituted water containing condensed chlorine: a very decided electro-magnetic effect was produced. A similar effect was produced by using nitro-muriatic acid, or aqua regia as it was formerly called, instead of the chlorine. The needle of the galvanometer in both cases turned round in the same direction as it does when zinc was substituted for the gold leaf and copper for the platina. Having tried, by the common method, the conducting powers of the diluted sulphuric acid and the water containing chlorine, I found that the diluted acid was the most powerful conductor. When the preceding experiment was repeated with discs of zinc and copper instead of discs of gold and platina, I found that the most powerful effect was produced when the diluted sulphuric acid was used. This experiment clearly proves that the interposed fluid does not act merely as a conductor to the electricity excited by

the imaginary electro-motive force, since in the first case the electricity generated is greatest when the conducting power of the fluid is least.

Experiment II.

Having made a small rectangular box divided into two equal compartments by a diaphragm of bladder, I introduced into one of them a disc of hard copper, and into the other an equal disc of soft copper. These discs being connected with the cups of the galvanometer, and the chambers filled with water, a considerable galvanic effect was produced, and the needle turned round as it does when the place of the hard copper was supplied with a disc of zinc. I then poured a little nitrous acid into the chamber containing the hard copper, and observed that the effect was diminished. By adding a little more acid the needle turned round several degrees in the opposite direction. This experiment completely overthrows the assumed principle that the galvanic effect increases with the conducting power of the fluid interposed between the metallic plates, since by increasing the conducting power of the fluid the effect was diminished, and by a proper increase was completely destroyed. It is a curious fact, that if nitric, sulphuric, or muriatic acid be used instead of the nitrous, the results will be quite the reverse.

Having thus, I trust, satisfactorily shown that the electric theory is founded on false principles, I shall now very shortly examine the truth of the most generally received chemical theory of galvanism.

3. Dr. Wollaston assumes that positive electricity is set at liberty by the combination of oxygen with one of the metals. This principle is frequently true, but in many cases it is totally false. This will be rendered obvious by the following experiments.

Experiment III.

Immerse two equal discs of zinc, connected by wires with the galvanometer, into the chambers of the rectangular box formerly used, and fill both compartments with water; no action will of course take place. Pour a little sulphuric, nitric, or muriatic acids into one of the chambers, a considerable galvanic effect will be produced, and the needle will turn in the same direction as it does when copper is substituted for the plate of zinc immersed in the chamber

containing the water alone. This agrees with the chemical theory. Again, instead of the above acids use nitrous acid, and the needle will turn round in the opposite direction. The same thing holds when discs of copper or iron are employed. This is completely at variance with the chemical theory, since that plate is negative, or corresponds with copper in the standard battery, on which the greatest chemical action of the fluid takes place. The following experiment is also hostile to the generally received theory.

Experiment IV.

Having taken two pieces of block tin, I cut the surface of one of them into ridges by means of a three-cornered file, so that the surface was doubled. With these two pieces I formed a binary combination, and immersed them in diluted nitro-muriatic acid; a very considerable electro-magnetic effect was produced, and the needle turned round in the same direction as it does when a plate of zinc is substituted for the plane disc in the standard battery. It is obvious that there must be a greater chemical action between the acid and the furrowed plate than the other, and yet the furrowed plate corresponds with copper in the standard battery, on which the least chemical action takes place. The results obtained in the following experiment were also unexpected.

Experiment V.

Take equal pieces of soft zinc, copper, iron, or brass, beat one of each pair on a smooth anvil till they are as hard as possible. Form a binary combination with pairs of the same metal, and use diluted sulphuric acid, and it will be found by the galvanometer that the hard metal in each case corresponds with zinc in the standard battery. If two pieces of steel be employed, one of them soft, and the other tempered, a galvanic effect will be produced, but of a contrary character. The soft steel will correspond with zinc, and the hard with copper, in the battery of comparison. The result of the following experiment seems also at variance with previous notions on the subject.

Experiment VI.

Having procured two small iron bars, with the ends made bright with a file, and copper wires connected with the other ends, I heated the end of one of

them, connected the wires with the galvanometer, and then immersed the hot and cold ends in water; a considerable action took place, and the cold iron was found to correspond with zinc in the standard battery. Since oxygen combines more rapidly with hot than with cold iron, positive electricity ought, according to the received opinions, to have appeared at the hot iron, whereas the contrary was actually the case. The following experiment is not only at variance with the theory of Dr. Wollaston, but seems also hostile to some of the generally received notions of chemists.

Experiment VII.

Let C be a cylinder of copper, about an inch in diameter, and two inches long, having a small copper tube *t* soldered in one end, whilst the other end is left open. Let Z be a small cylinder of zinc, having a copper wire *w* soldered to the lower end. The wire, being covered with a thread and passed through the tube, is firmly cemented with electric cement, metallic contact being carefully avoided. Another end having a strong brass tube with an internal screw is now soldered in the top of the copper cylinder. The interior surface of the cylinder of zinc is covered with electric cement to prevent the acid acting on it. The whole is now nearly filled with water, and a little sulphuric acid is introduced into the zinc cylinder by means of a very slender glass funnel. The whole is now completely filled with water, and a solid screw dipped in electric cement, and screwed into the top of the brass tube, whilst it is heated, renders the whole completely air-tight. The acid is now to be mixed with the water by frequently inverting and shaking the cylinder. If the copper and zinc cylinders be connected with the galvanometer, the battery will continue to act for a day or two with the same energy as if the whole had been left exposed to the air. As there is no room for the disengagement of hydrogen, the oxygen of the water cannot combine with the zinc to convert it into an oxide; nevertheless chemical action goes on, and the zinc is dissolved in the acid. From this experiment it is obvious that the oxidation of the zinc and the combination of nascent hydrogen with the electric fluid, as Dr. Bostock supposes, has nothing to do with the production or transfer of the electricity which appears at the surface of the zinc. The metal is still, however, dissolved or reduced from

a solid to a fluid state; and as its capacity for caloric has undergone a change, may not its capacity for the electric fluid have also undergone a certain change? Hence it is possible that the true theory of galvanism may be more intimately connected with that of latent heat than has yet been supposed. Since the zinc is dissolved without the assistance of oxygen from the water, it appears that the atoms of the acid have combined with the pure brilliant atoms of the metal, without the necessity of the metal being first converted to an oxide.

From the short view that I have taken of this interesting subject, it appears that the electric theory is quite unfounded, and that the chemical theory will require some modification to embrace the facts contained in the last experiments. This I shall not, however, attempt at present; as my object in this paper is rather to demolish old fabrics and collect new materials, from which a more substantial edifice may be raised.

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MDCCCXXIX.

AMENDED STATUTES.

Extracts from the MINUTES of COUNCIL.

(*Printed by Order of the President and Council.*)

June 26, 1828.—The following alteration in the Statutes, having been duly proposed, was put to the Ballot, and carried:—

That at the end of the 3rd Section of the 1st Chapter of the Statutes, which relates to the Election of a Prince of the Blood Royal, or a Peer of the United Kingdom, or one of His Majesty's Privy Council, or for a Prince, or the son of a Sovereign Prince, be added the following words:—viz.

"Notice of such intention having been publicly announced by a Member at the preceding Meeting of the Society."

June 29, 1829.—The following alteration in the Statutes, having been duly proposed at a former Meeting, was this day declared by Ballot in the Affirmative; namely, Sections II, III, IV and V of Chapter III, were repealed, and the following Sections substituted, leaving Section I of the same Chapter unaltered.

SECTION II.

"Every person elected a Fellow on the Home List, shall, besides the Admission Money, further contribute to the use of the Society, previous to his Admission, the sum of *Forty Pounds*, excepting in cases wherein the Council shall grant leave for the substitution of a payment after the rate of *One Pound per Quarter* to the Society as long as he shall continue a Fellow thereof; such payments to commence and become payable on the quarter day next succeeding the time of his election."

SECTION III.

"Every Fellow paying Annual Contributions, may at any time compound for his future contributions, by paying a sum equal to ten times the amount of his annual contribution."

SECTION IV.

"For greater convenience in receiving and collecting the periodical contributions from Fellows, the same shall be paid yearly to the twenty-fifth day of March; and in proportion for a part of a year, by any person elected after the beginning, or withdrawing from the Society before the end, of the year."

SECTION V.

"Every Fellow of the Society liable to the payment of Annual Contributions, shall, previously to the twenty-fifth day of March in every year, bring or send to the Treasurer, or his Deputy, his yearly Contribution, or such proportion of it as shall be then due. And if any such Fellow shall fail to bring or send in the same as aforesaid before the first day of May next following, his name shall be suspended in the Public Meeting Room of the Society, as being in arrear of Contribution, and shall continue so suspended until the Contribution so due be paid. And if any such Fellow shall fail to bring or send in his arrears for the preceding year on or before the Meeting of the Society next preceding St. Andrew's Day, and if no satisfactory reason be then assigned to the President and Council for the non-payment of such arrears, he shall cease to be a Fellow of the Society. Provided, nevertheless, that on a solicitation for re-admission being addressed to the President and Council, by an individual so circumstanced, within the space of three months following St. Andrew's Day, the case of the individual so soliciting shall be stated by the President from the Chair at one of the ordinary Meetings of the Society, and the question of his re-admission decided by Ballot, according to the majority of votes, at the next Meeting of the Society."

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RIGAUD (S. P.) Astronomical Observations made at the Radcliffe Observatory, Oxford, from May 1, 1828, to May 1, 1829. By, and under the direction of, Stephen Peter Rigaud, Esq. F.R.S. (*M.S.*) Fol.	Professor Rigaud, F.R.S.
ROBINSON (P. F.) The New Vitruvius Britannicus. Part I. containing the Antiquities of Woburn Abbey. Imp. Fol. *Lond.* 1829.	P. F. Robinson, Esq.
ROGER (M.) Recherches Philosophiques sur la Langue Ouolofe, suivies d'un Vocabulaire abrégé Français-ouolof. 8vo. *Paris* 1829.	The Baron Roger.
ROGET (P. M.) Description of the New Sliding Rule of Involution, invented by Dr. Roget, with Instructions for using it. 8vo. *Lond.* 1828.	Dr. Roget, Sec. R.S.
ROSS (Capt. John). Treatise on Navigation by Steam, comprising a History of the Steam Engine, and an Essay towards a System of the Naval Tactics peculiar to Steam Navigation, as applicable both to Commerce and Maritime Warfare; including a Comparison of its Advantages as related to other Systems in the Circumstances of Speed, Safety, and Economy, but more particularly in that of the National Defence. Illustrated with Plates and Engravings. 4to. *London* 1828.	Captain John Ross, R.N.
SANTOS. Nova Theoria do Universo. Fundada nas notaveis Differenças de Densidades, e Naturezas Constituintes dos Corpos Celestes; donde resultão a fluctuação dos Satellites, e a anti gravitação do Sol, consideradas como Leis Universaes; e se excluem as attracções, as forças de projecção, velocidades iniciaes, vacuos, &c. Para servir de Introducçao a outras Obras Astronomicas, Geologicas, Physicas, e de Historia Natural. 8vo. *Rio de Janeiro* 1827.	Señor José Victorino dos Santos e Sousa.
SERULLAS (M.) Sur un Nouveau Composé de Brome et de Carbone, et sur les Iodures de Carbone. 8vo. *Paris.*	M. Serullas.
——— Sur les Bromures d'Arsenic et de Bismuth, et sur le Bromure d'Antimoine. 8vo. *Paris* 1827.	———
——— De l'Action de l'Acide Sulfurique sur l'Alcool, et des Produits qui en resultent. 8vo. *Paris* 1828.	———
SEYFFARTH (G.) Brevis Defensio Hieroglyphices inventæ à F. A. G. Spohn et G. Seyffarth. Scripsit G. Seyffarth. 4to. *Lipsiæ* 1827.	Dr. Seyffarth.
——— Remarks upon an Egyptian History, in Egyptian characters, in the Royal Museum at Turin, with reference to an Article in the Edinburgh Review. By Dr. G. Seyffarth, Prof. Univ. Leipzig. 8vo. *Lond.* 1828.	———

Presents.	Donors.
SKIADAN (M.) Description d'un Eriomètre, ou Instrument pour mesurer la Finesse de la Laine, de l'Invention de M. Skiadan; publiée par M. le Doct. Hamel. 8vo. *Moscow* 1828.	Dr. Hamel.
SOANE (J.) Designs for completing some of the Public Buildings at Westminster, and for correcting Defects in others. By J. Soane, Architect. Fol. *London* 1829.	John Soane, Esq. F.R.S.
SOEMMERRING, *see* THILO.	
SOUTH (J.) Reply to a Letter in the Morning Chronicle relative to the Interest which the British Government evinces in the promotion of Astronomical Science. 8vo. *Lond.* 1829.	James South, Esq. F.R.S.
——— Return to an Order of the House of Commons for Memorials or Reports presented to the Government since Jan. 1828, on the Nautical Almanac and Board of Longitude; and for a Return of the Expenses of the late Board of Longitude.	———
——— Refutation of the numerous Mis-statements and Fallacies contained in a Paper presented to the Admiralty. 8vo. *Lond.* 1829.	———
STANHOPE (Earl.) An Address delivered at the Anniversary Meeting of the Medico-Botanical Society. 8vo. *Lond.* 1829.	The Earl Stanhope, F.R.S.
SWAINSON (W.) Zoological Illustrations. The Drawings and Descriptions by William Swainson, Esq. F.R.S. No. 1 of Second Series. 8vo. *Lond.* 1829.	William Swainson, Esq. F.R.S.
TAYLOR (John.) Records of Mining. Part I. John Taylor, F.R.S. 4to. *Lond.* 1829.	John Taylor, Esq. F.R.S.
THILO (L.) De Tabulis Iconographicis, quibus Maculæ Solis, mensibus Anni 1826 sex posterioribus, et Anni 1827 sex prioribus, à Viro illustrissimo S. Th. à Soemmerring observatæ, adumbrantur. Dissertatio Ludovici Thilo, Phil. Doct. 4to. *Francof. ad Mæn.* 1828.	Dr. Soemmerring, F.R.S.
THURY (M. de.) Programme d'un Concours pour le Percement de Puits Forés suivant la Méthode Artésienne. 8vo. *Paris* 1828.	The Viscount de Thury.
TIARKS (J. L.) Report on Astronomical Observations for ascertaining the most North-western Point of the Lake of the Woods. 8vo. *New York* 1827.	Dr. Tiarks, F.R.S.
VETUSTA MONUMENTA. Vol. V. Plates LI.—LX.	The Society of Antiquaries.
WATKINS (F.) A Popular Sketch of Electro-Magnetism, or Elector-Dynamics. 8vo. *Lond.* 1828.	Mr. F. Watkins.
WOODLEY (W.) The Universe as it is. With Engravings designed by W. Woodley. 8vo. *Lond.* 1829.	William Woodley, Esq.
YARRELL (W.) Papers on the White Bait and Shad, on the Tapir of America, and on the Anatomy of the Corvorant. 8vo. *Lond.* 1828.	William Yarrell, Esq.
YORKSHIRE PHILOSOPHICAL SOCIETY. The Annual Report of the Yorkshire Philosophical Society for 1828. 8vo.	The Society.

METEOROLOGICAL JOURNAL,

KEPT BY THE ASSISTANT SECRETARY

AT THE APARTMENTS OF THE

ROYAL SOCIETY,

BY ORDER OF

THE PRESIDENT AND COUNCIL.

MDCCCXXIX.

METEOROLOGICAL JOURNAL FOR JANUARY, 1829.

1829. January.	9 o'clock, A.M.		3 o'clock, P.M.		Dew Point at 9 A.M. in degrees of Fahr.	External Thermometer.				Rain, in inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	Remarks
						Fahrenheit.		Self-registering.				
	Barom.	Attach. Therm.	Barom.	Attach. Therm.		9 A.M.	3 P.M.	Lowest.	Highest.			
☿ 1	29.854	44.0	29.822	46.8	40	40.8	45.7	37.2	45.7	0.008	WSW	Fine and clear—light brisk wind.
♀ 2	29.860	43.2	29.917	45.1	37	38.7	43.0	36.8	43.0		NNW	Fine—faint haze, and light wind.
♄ 3	29.916	42.8	29.921	43.8	38	38.8	41.5	35.3	41.5		WSW	A.M. Strong haze. P.M. Fine.
☉ 4	29.578	43.7	29.453	45.7	42	42.2	42.8	37.9	43.7	0.219	WSW	A.M. Foggy—rain. P.M. Cloudless—brisk wind.
● ☽ 5	29.704	40.2	29.814	40.3	34	35.8	36.0	34.4	36.7	0.008	N	Fine and cloudless—light wind.
♂ 6	29.976	35.8	29.973	38.0	31	31.0	35.2	29.3	35.2		N	A.M. Cloudless—light wind. P.M. Hazy. Snow at night.
☿ 7	30.011	37.2	29.951	38.6	34	35.4	37.3	30.2	37.3		NNE	A.M. Foggy. P.M. Fine.
♃ 8	29.872	36.7	29.856	37.3	32	33.3	34.7	31.8	34.7		N	Foggy. P.M. Snow.
♀ 9	29.795	37.6	29.707	39.0	33	34.8	36.7	32.5	36.7		N	Foggy—light wind.
♄ 10	29.624	38.7	29.656	39.9	35	36.0	38.3	33.8	38.3		N	Light clouds—foggy.
☉ 11	29.878	38.3	29.910	37.3	30	30.6	31.3	29.8	32.6		E	Overcast and hazy.
☽ 12	29.989	37.4	29.971	37.8	32	36.7	36.2	29.3	36.7		N	Overcast and hazy.
♂ 13	30.090	37.2	30.076	38.8	32	35.9	37.8	34.0	37.8		E	Overcast and foggy.
☿ 14	30.049	37.7	30.026	38.9	34	34.7	37.3	32.7	37.3		N	Lightly overcast and foggy. Snow at noon.
♃ 15	29.924	38.8	29.809	39.7	35	36.3	37.4	34.0	37.4	0.014	NNE	Overcast and foggy.
♀ 16	29.631	37.4	29.609	36.3	25	32.2	30.2	31.0	32.2		NNE	Overcast and foggy.
♄ 17	29.762	33.7	29.807	35.2	21	30.0	32.0	27.3	32.0		N	Overcast and foggy.
☉ 18	29.967	33.5	29.999	34.7	25	27.8	31.7	25.8	31.7		N	Dense fog—dark.
☽ 19	30.130	32.8	30.117	33.7	25	27.7	33.7	25.7	33.7		N	A.M. Very dense fog. P.M. Fine and clear. Evening foggy.
○ ♂ 20	30.032	31.3	29.971	33.6	24	27.7	31.4	22.7	31.4		N	A.M. Fog and hoar frost. P.M. Fine—strong haze.
☿ 21	29.893	32.7	29.829	32.6	22	28.4	29.3	26.3	29.3		E	Overcast and foggy. Light snow.
♃ 22	29.673	29.7	29.635	30.6	18	25.2	29.5	22.3	29.5		ESE	Fine and cloudless—light wind.
♀ 23	29.547	29.8	29.489	28.7	16	25.3	22.8	23.4	25.3		E	Light clouds and haze. Snow P.M.
♄ 24	29.524	25.5	29.559	28.6	22	22.7	27.7	19.5	27.7		NNW	Overcast and hazy—light wind. Snow.
☉ 25	29.641	27.6	29.606	28.3	22	22.6	24.4	21.3	33.0		W	A.M. Overcast. P.M. Fine & cloudless.
☽ 26	29.041	30.8	29.100	34.4	35	35.2	43.2	20.7	43.2	0.042	E	A.M. Foggy.
♂ 27	29.113	36.7	29.131	39.7	38	38.3	41.4	34.2	41.4		S	Fine—lightly cloudy.
☿ 28	29.377	37.7	29.517	40.8	35	35.1	41.6	33.4	41.6		SW	A.M. Cloudless. P.M. Overcast and hazy.
♃ 29	29.547	37.7	29.473	40.8	34	35.2	40.3	32.7	40.3		NE	Overcast. Strong fog A.M.
♀ 30	29.596	39.4	29.664	42.2	35	34.8	40.4	32.2	40.4		N	Lightly overcast. Fine at noon.
♄ 31	30.141	40.3	30.230	38.7	37	37.4	38.7	34.1	38.7		N	Cloudy—light wind.
	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Sum		
	29.763	36.3	29.761	37.6	30.7	33.1	35.8	30.1	36.3	0.291		

Monthly Mean of the Barometer, corrected for Capillarity and reduced to 32° Fahr. ..... { 9 A.M. 29.755 | 3 P.M. 29.750 }

OBSERVANDA.

Height of the Cistern of the Barometer above a Fixed Mark on Waterloo Bridge..... = 83 feet 2½ in.

..... above the mean level of the Sea (presumed about) ..... = 95 feet.

The External Thermometer is 2 feet higher than the Barometer Cistern.

Height of the Receiver of the Rain Gauge above the Court of Somerset House ..... = 79 feet 0 in.

The hours of observation are of Mean Time, the day beginning at Midnight.

The Thermometers are graduated by Fahrenheit's Scale.

The Barometer is divided into inches and decimals.

METEOROLOGICAL JOURNAL FOR FEBRUARY, 1829.

1829. February.	9 o'clock, A.M. Barom.	9 o'clock, A.M. Attach. Therm.	3 o'clock, P.M. Barom.	3 o'clock, P.M. Attach. Therm.	Dew Point at 9 A.M. in degrees of Fahr.	External Thermometer. Fahrenheit. 9 A.M.	Fahrenheit. 3 P.M.	Self-registering. Lowest.	Self-registering. Highest.	Rain, in inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	Remarks.
☉ 1	30.449	36.3	30.474	38.3	24	32.3	35.7	29.3	35.7		N	Lightly cloudy—light wind.
☽ 2	30.575	33.3	30.565	35.8	22	25.7	33.4	23.2	33.4		ESE	Fine and cloudless.
♂ 3	30.593	32.4	30.566	35.3	24	27.0	34.8	23.9	34.8		S	A.M. Fog and hoar frost. P.M. Fine and cloudless.
● ☿ 4	30.417	34.8	30.321	36.7	32	34.4	38.3	25.7	43.6		SSW	Foggy—rain.
♃ 5	30.145	39.7	30.230	42.8	44	44.5	43.7	33.6	44.5	0.125	NNW	Cloudy—light wind.
♀ 6	30.341	42.7	30.385	45.7	39	39.0	43.7	37.5	43.7		W	Foggy. P.M. Very light rain.
♄ 7	30.163	44.7	30.129	46.7	42	42.8	46.3	38.3	46.3		W	Overcast and foggy.
☉ 8	30.312	44.2	30.381	43.8	41	41.3	39.4	40.1	41.3		N	Overcast. Foggy A.M.
☽ 9	30.325	42.2	30.290	44.4	34	39.2	43.5	34.0	43.5		W	A.M. Foggy. P.M. Fine—cloudy—light wind.
♂ 10	30.426	42.5	30.428	42.2	35	37.6	38.8	36.5	41.0		E	Overcast.
☿ 11	30.339	43.7	30.300	46.2	42	42.3	47.3	36.7	47.3		SW	Overcast and foggy—light rain.
♃ 12	30.273	46.2	30.221	48.2	42	42.8	45.8	41.2	46.3	0.050	S	Overcast and foggy—light rain.
♀ 13	30.181	49.6	30.163	50.5	45	45.3	47.8	41.8	47.8	0.014	W	A.M. Foggy. P.M. Fine—light wind.
♄ 14	30.175	47.7	30.149	50.4	40	43.7	48.3	42.1	48.3		W	Lightly cloudy—hazy.
☉ 15	30.108	48.3	30.086	51.3	45	45.3	49.4	43.3	49.7		WSW	A.M. Fine. P.M. Foggy—very light rain.
☽ 16	30.026	49.3	29.949	51.8	44	46.8	50.4	44.5	50.8	0.008	SW	Fine—lightly overcast.
♂ 17	29.801	47.7	29.834	50.3	42	42.7	47.7	40.2	47.7		WSW	Lightly overcast and hazy.
○ ☿ 18	29.886	46.3	29.860	46.8	38	38.3	37.5	36.0	38.7		SE	A.M. Fine & cloudless. P.M. Overcast.
♃ 19	29.646	43.7	29.569	47.7	37	39.6	49.4	35.3	49.4		ESE	Fine—lightly cloudy.
♀ 20	29.641	48.3	29.620	50.3	44	44.9	49.6	38.8	49.6		SSW	Fine—cloudy.
♄ 21	29.198	48.8	29.122	51.3	47	47.5	50.7	42.7	50.7		SSE	Cloudy—light rain.
☉ 22	29.147	48.8	29.244	49.4	46	46.3	43.6	43.7	46.3	0.156	WNW	Overcast and foggy.
☽ 23	29.529	43.8	29.525	45.8	28	37.4	39.3	36.3	39.3		E	A.M. Cloudy. P.M. Fine and clear. Light wind.
♂ 24	29.434	41.5	29.448	43.2	35	35.7	39.7	32.5	39.7		E	Overcast and foggy.
☿ 25	29.905	41.3	29.998	44.3	35	38.3	41.2	34.8	41.3		ESE	Overcast and foggy.
♃ 26	30.098	42.8	30.028	43.9	39	39.3	40.3	35.7	41.3		ESE	Overcast and foggy. P.M. Rain.
♀ 27	30.118	45.3	30.221	46.3	41	41.7	41.6	38.4	41.7	0.389	NNE	Overcast and foggy—light wind.
♄ 28	30.385	42.7	30.335	44.0	29	34.8	39.2	32.3	39.2		ESE	Fine and cloudless—light wind.
	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Sum		
	30.058	43.5	30.048	45.5	37.7	39.9	43.1	36.4	43.7	0.742		

Monthly Mean of the Barometer, corrected for Capillarity and reduced to 32° Fahr. ..... { 9 A.M. 30.032 } { 3 P.M. 30.018 }

OBSERVANDA.

Height of the Cistern of the Barometer above a Fixed Mark on Waterloo Bridge..... = 83 feet 2¼ in.

.....above the mean level of the Sea (presumed about) ..... = 95 feet.

The External Thermometer is 2 feet higher than the Barometer Cistern.

Height of the Receiver of the Rain Gauge above the Court of Somerset House..... = 79 feet 0 in.

The hours of observation are of Mean Time, the day beginning at Midnight.

The Thermometers are graduated by Fahrenheit's Scale.

The Barometer is divided into inches and decimals.

METEOROLOGICAL JOURNAL FOR MARCH, 1829.

1829. March.	9 o'clock, A.M. Barom.	9 o'clock, A.M. Attach. Therm.	3 o'clock P.M. Barom.	3 o'clock P.M. Attach. Therm.	Dew Point at 9 A.M. in degrees of Fahr.	External Thermometer. Fahrenheit. 9 A.M.	3 P.M.	Self-registering. Lowest.	Highest.	Rain, in inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	Remarks.
☉ 1	30.173	36.3	30.131	38.7	32	32.7	33 3	27.0	34.7		E	A.M. Fine—light wind. P.M. Overcast.
☽ 2	30.173	36.9	30.204	38.6	33	35.0	36.7	31.2	35.7		ENE	Overcast and foggy.
♂ 3	30.338	37.7	30.331	39.3	35	35.6	37.2	32.8	40.3		NNE	Overcast and foggy.
☿ 4	30.167	40.7	30.133	42.8	41	41.1	41.9	34.7	42.3		NNE	Cloudy—light wind.
♃ 5	30.215	39.3	30.156	42.6	36	36.8	42.7	32.3	42.7		ENE	A.M. Foggy. P.M. Fine—lightly cloudy. Light wind.
♀ 6	30.090	43.6	30.031	46.0	41	41.6	43.5	36.2	44.6		N	Overcast and foggy.
♄ 7	29.987	43.7	29.981	46.7	41	41.0	45.7	38.7	45.7		N	Overcast and foggy—light wind.
☉ 8	29.993	45.7	29.980	47.8	44	45.4	46.0	40.3	46.0		NNE	Cloudy and foggy. Light rain A.M.
☽ 9	29.883	45.2	29.832	47.6	41	41.7	48.3	40.2	48.3	0.036	W	Lightly cloudy and foggy.
♂ 10	29.872	44.4	29.859	47.3	36	40.0	45.0	37.0	45.0		N	A.M. Cloudless. P.M. Cloudy—light wind.
☿ 11	29.850	42.9	29.804	45.2	36	38.7	42.3	32.5	42.3		NNE	Overcast and hazy—light wind.
♃ 12	29.763	41.7	29.703	44.7	32	37.8	39.9	33.7	41.3		NE	A.M. Overcast. P.M. Fine and clear. Light wind.
♀ 13	29.644	42.3	29.703	45.0	38	38.7	43.5	34.6	43.5		NNE	Lightly cloudy—brisk wind.
♄ 14	29.873	41.8	29.867	44.3	24	37.6	41.3	34.3	41.3		NNW	A.M. Fine. P.M. Overcast—light wind.
☉ 15	29.916	39.4	29.892	41.2	27	33.7	39.8	26.8	39.8		WSW	Fine—light clouds and haze.
☽ 16	29.775	37.8	29.662	41.7	29	34.6	40.3	27.8	40.3		E	Clear and cloudless—light wind.
♂ 17	29.497	38.7	29.476	42.8	29	37.7	46.5	28.7	46.5		S	Fine—lightly cloudy.
☿ 18	29.710	40.7	29.731	47.3	41	41.4	51.3	33.0	52.4		SSE	Lightly overcast and foggy. Fine at noon.
♃ 19	29.657	48.7	29.573	53.9	43	51.4	58.3	40.6	58.4	0.014	SSE	Fine—light clouds—brisk wind. Light rain early A.M.
○ ♀ 20	29.668	55.2	29.796	57.4	51	53.4	57.6	50.2	58.6		SW	Fine and clear—light brisk wind.
♄ 21	30.114	54.3	30.094	57.3	44	49.8	55.3	45.7	55.4		E	Lightly cloudy—light wind.
☉ 22	29.917	49.6	29.825	53.7	45	45.3	48.6	38.4	51.7		ENE	Fine—lightly cloudy—light wind.
☽ 23	29.817	51.0	29.884	52.7	41	47.6	47.1	42.8	49.4		E	Fine—light clouds—brisk wind.
♂ 24	29.912	46.6	29.876	48.6	32	40.3	44.7	37.5	44.7		NNE	Fine and clear—light wind.
☿ 25	29.939	41.1	29.910	45.3	30	36.8	45.7	27.6	45.7		ENE	Fine and cloudless—light brisk wind.
♃ 26	29.983	44.3	29.965	47.2	31	42.2	45.7	34.8	45.7		NNE	Lightly cloudy.
♀ 27	29.952	46.8	29.872	48.9	28	42.0	45.7	37.7	45.7		ESE	Clear and cloudless.
♄ 28	29.660	44.3	29.531	48.7	39	42.9	50.7	33.6	50.7		E	Fine and cloudless—light wind.
☉ 29	29.290	46.6	29.279	47.8	44	44.5	46.3	41.6	46.3	0.542	NNE	Thickly overcast and foggy. Heavy rain at 1 A.M. and afterwards.
☽ 30	29.231	46.3	29.202	48.6	42	42.8	46.7	39.8	47.2		NNE	Overcast and foggy.
♂ 31	29.221	46.0	29.230	48.0	41	41.7	44.3	39.7	44.3		NE	Overcast—light wind.
	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Sum		
	29.848	43.9	29.823	46.7	37.0	41.0	45.2	35.9	45.7	0.592		

Monthly Mean of the Barometer, corrected for Capillarity and reduced to 32° Fahr. ..... { 9 A.M. 29.821 | 3 P.M. 29.790 }

OBSERVANDA.

Height of the Cistern of the Barometer above a Fixed Mark on Waterloo Bridge ..... = 83 feet 2½ in.

..... above the mean level of the Sea (presumed about) ..... = 95 feet.

The External Thermometer is 2 feet higher than the Barometer Cistern.

Height of the Receiver of the Rain Gauge above the Court of Somerset House ..... = 79 feet 0 in.

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The Barometer is divided into inches and decimals.

METEOROLOGICAL JOURNAL FOR APRIL, 1829.

1829. April.	9 o'clock, A.M. Barom.	9 o'clock, A.M. Attach. Therm.	3 o'clock P.M. Barom.	3 o'clock P.M. Attach. Therm.	Dew Point at 9 A.M. in degrees of Fahr.	External Thermometer. Fahrenheit. 9 A.M.	Fahrenheit. 3 P.M.	Self-registering. Lowest.	Self-registering. Highest.	Rain, in inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	Remarks.
☿ 1	29.416	44.3	29.426	47.5	31	39.8	45.3	34.7	45.8		NNW	Lightly cloudy.
♃ 2	29.507	41.9	29.547	45.7	32	36.4	43.6	30.7	45.7	0.047	NW	Fine—cloudy. Light snow at 7 A.M.
● ♀ 3	29.678	44.2	29.699	48.4	31	39.6	48.3	31.8	48.3		NW	Fine—light clouds.
♄ 4	29.700	48.3	29.609	51.7	41	47.8	51.3	36.8	52.3		S	A.M. Clear. P.M. Soft clouds. Evening rainy. Light wind.
☉ 5	29.375	50.7	29.280	53.6	44	49.0	52.7	43.8	53.6	0.034	SSE	Fine and clear—light broken clouds. Brisk wind.
☽ 6	29.179	52.0	29.121	54.8	45	50.3	51.7	44.2	53.8		S	Soft broken clouds. Evening rainy.
♂ 7	29.161	49.6	29.203	52.3	44	44.8	50.3	41.6	51.2	0.161	SW	A.M. Thickly overcast. P.M. Fine and clear. Evening rainy.
☿ 8	29.418	49.0	29.411	52.5	43	46.0	49.2	37.6	51.2	0.056	S	Lightly cloudy—showery.
♃ 9	29.906	47.8	29.922	50.3	43	43.1	46.2	37.5	47.2	0.367	ESE	Cloudy and foggy. Rain A.M.
♀ 10	29.450	52.6	29.476	55.3	43	48.5	52.0	39.8	53.3	0.172	WSW	Fine—lightly cloudy.
♄ 11	29.632	52.6	29.593	55.7	40	47.8	53.7	38.8	56.1		SW	Fine—lightly overcast. Rain at night.
☉ 12	29.237	53.3	29.211	57.3	51	51.8	56.8	47.3	57.6	0.158	S	Brisk wind. A.M. Lowering. P.M. Clear. Evening rainy.
☽ 13	29.143	56.7	29.159	55.6	48	54.4	51.3	48.5	55.3	0.055	SSW	A.M. Fine—brisk wind. P.M. Rain.
♂ 14	29.291	55.3	29.096	57.2	46	51.7	54.1	42.7	57.3	0.136	SSE	A.M. Fine. P.M. Rain. Evening heavy rain and brisk wind.
☿ 15	29.137	56.4	29.246	56.6	43	50.5	54.0	47.4	54.4	0.217	S var.	Strong wind A.M. Lowering. P.M. Fine. At noon heavy rain and boisterous wind.
♃ 16	29.231	52.3	29.281	52.5	47	47.3	47.6	44.7	49.7	0.033	NNE	Overcast and foggy—rain.
♀ 17	29.636	53.7	29.706	55.2	49	49.7	54.2	39.4	56.3	0.152	WSW	Fine—soft clouds. Evening showery.
♄ 18	29.834	54.3	29.815	56.3	45	51.8	55.7	44.8	59.2	0.028	WSW	Fine—light clouds and wind.
○ ☉ 19	29.699	55.3	29.636	56.3	42	51.2	51.3	42.3	59.3		WSW	A.M. Fine. P.M. Lowering—rain—strong wind.
☽ 20	29.742	51.3	29.820	54.6	40	47.7	53.0	39.7	53.3	0.150	NW	Cloudy.
♂ 21	29.840	51.7	29.691	54.7	41	50.7	53.5	39.3	56.4		ESE	A.M. Cloudy. P.M. Clear—brisk wind.
☿ 22	29.480	51.3	29.474	51.9	47	47.4	47.6	45.9	47.7		E	Overcast—light rain—brisk wind.
♃ 23	29.692	51.2	29.725	55.2	43	48.3	56.6	44.4	56.7	0.033	E	A.M. Overcast. P.M. Fine.
♀ 24	29.742	51.1	29.792	52.8	48	48.3	48.7	44.7	50.0		NW	Cloudy.
♄ 25	29.792	49.7	29.896	47.2	46	45.8	40.3	43.5	46.7	0.200	N	Rain—light wind.
☉ 26	30.118	50.2	30.088	51.7	35	46.8	50.5	35.4	51.4	0.213	E	Fine—lightly cloudy.
☽ 27	29.775	48.5	29.585	50.8	39	45.3	51.0	39.5	51.7		SSW	A.M. Fine. P.M. Showery.
♂ 28	29.675	49.4	29.315	50.7	38	44.7	50.7	39.5	52.3	0.094	WSW	A.M. Overcast. P.M. Fine and clear—violent wind.
☿ 29	29.687	47.6	29.757	49.4	29	43.7	43.7	37.8	46.3		W	A.M. Fine—brisk wind. P.M. Overcast.
♃ 30	29.906	44.7	29.873	48.7	27	44.7	45.8	34.2	52.2		NNW	Cloudy.
	Mean 29.545	Mean 50.6	Mean 29.525	Mean 52.7	Mean 41.4	Mean 47.2	Mean 50.4	Mean 40.6	Mean 52.4	Sum 2.286		

Monthly Mean of the Barometer, corrected for Capillarity and reduced to 32° Fahr. ..... { 9 A.M. 29.502 } { 3 P.M. 29.477 }

OBSERVANDA.

Height of the Cistern of the Barometer above a Fixed Mark on Waterloo Bridge..... = 83 feet 2½ in.

..... above the mean level of the Sea (presumed about) ..... = 95 feet.

The External Thermometer is 2 feet higher than the Barometer Cistern.

Height of the Receiver of the Rain Gauge above the Court of Somerset House ..... = 79 feet 0 in.

The hours of observation are of Mean Time, the day beginning at Midnight.

The Thermometers are graduated by Fahrenheit's Scale.

The Barometer is divided into inches and decimals.

METEOROLOGICAL JOURNAL FOR MAY, 1829.

1829. May.	9 o'clock, A.M. Barom.	9 o'clock, A.M. Attach. Therm.	3 o'clock, P.M. Barom.	3 o'clock, P.M. Attach. Therm.	Dew Point at 9 A.M. in degrees of Fahr.	External Thermometer. Fahrenheit. 9 A.M.	3 P.M.	Self-registering. Lowest.	Highest.	Rain, in inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	Remarks.
♀ 1	29.641	49.9	29.654	53.9	47	52.7	56.7	44.3	57.3		NNE	Fine—lightly cloudy—light wind.
♄ 2	29.690	56.7	29.678	57.7	45	56.5	58.8	46.8	61.3		WSW	Fine and clear—lightly cloudy.
● ☉ 3	29.702	56.3	29.610	58.8	50	55.0	57.6	47.8	59.4		WSW	Soft broken clouds—light wind. Evening rainy.
☽ 4	29.822	58.3	29.921	59.4	37	52.8	59.7	46.3	61.2	0.019	NNW	Fine and clear—light clouds.
♂ 5	30.044	59.0	30.029	59.0	46	56.6	58.6	45.3	60.3		S	Fine—diffused clouds. Strong wind.
☿ 6	29.962	57.6	29.973	61.8	55	54.7	62.3	53.6	62.7		WSW	Overcast—broken clouds. Light deposition A.M.
♃ 7	30.007	58.7	29.992	60.7	51	55.3	58.3	44.4	59.7		W	A.M. Lowering. P.M. Fine—nearly cloudless.
♀ 8	30.185	61.8	30.161	62.7	53	58.7	63.6	45.3	64.6		E	Fine—cloudy.
♄ 9	30.163	60.7	30.114	63.3	49	57.7	63.5	48.4	64.4		NW	Fine—light clouds.
☉ 10	30.092	64.3	30.052	65.4	52	60.6	66.5	49.7	69.2		W	Clear and cloudless—light breeze.
☽ 11	30.087	65.7	30.011	65.3	52	60.0	62.3	51.5	62.8		ESE	Fine—diffused clouds—light wind.
♂ 12	29.978	65.7	29.941	65.8	51	61.3	61.7	52.5	65.2		E	Fine and clear—light brisk wind.
☿ 13	29.978	65.6	29.954	64.7	50	61.3	63.7	46.8	64.6		ESE	Clear and cloudless—light wind.
♃ 14	29.990	63.4	29.933	65.7	51	58.5	68.7	48.6	69.7		W	Fine and clear—light clouds.
♀ 15	30.001	68.2	29.982	67.6	47	62.7	69.5	51.7	70.5		NNW	Fine—light clouds.
♄ 16	30.094	61.0	30.095	64.3	55	55.7	60.4	52.4	61.0	0.097	E	A.M. Overcast. P.M. Fine.
☉ 17	30.171	66.4	30.137	65.7	51	60.8	64.8	47.7	65.4		E	Fine and cloudless—light wind.
○ ☽ 18	30.081	65.7	30.018	64.5	49	61.7	62.7	46.2	63.7		ESE	Fine and cloudless—light wind.
♂ 19	29.925	66.7	29.908	65·5	54	62.3	65.3	47.6	66.7		E	Clear and cloudless—light breeze.
☿ 20	29.961	65.3	29.957	66.3	49	62.3	69.0	50.5	69.8		E	Clear and cloudless—brisk wind.
♃ 21	30.117	69.7	30.091	68.3	50	64.9	68.6	51.3	70.6		NE	Fine and clear—light clouds—brisk wind.
♀ 22	30.163	62.7	30.135	65.8	47	55.5	67.9	44.7	69.3		N	Fine and clear—light clouds and wind.
♄ 23	30.232	69.0	30.185	69.0	54	62.8	72.9	49.4	74.7		NNE	Fine—light clouds.
☉ 24	30.220	74.2	30.184	68.7	52	65.7	60.0	52.3	68.8		W	A.M. Broken clouds. P.M. Rain. Evening Fine.
☽ 25	30.409	62.3	30.414	62.3	46	53.4	57.6	46.6	58.2	0.197	NNE	Lightly cloudy—brisk wind.
♂ 26	30.395	58.8	30.397	62.8	44	54.0	63.7	46.9	64.2		NNE	Brisk wind. A.M. Cloudy. P.M. Cloudless.
☿ 27	30.292	64.7	30.248	64.2	49	59.6	66.8	46.7	67.7		N	Clear and cloudless—brisk wind.
♃ 28	30.219	66.3	30.169	66.2	53	62.3	69.3	48.8	69.7		NNE	Clear and cloudless—light brisk wind.
♀ 29	30.191	61.7	30.201	65.7	53	57.5	64.0	49.4	65.2		N	Cloudy—brisk wind.
♄ 30	30.189	59.7	30.183	62.8	44	56.3	57.5	49.6	59.4		N	Interfused clouds—light wind.
☉ 31	30.212	62.7	30.163	63.8	46	56.0	63.0	46.2	64.2		W	Lightly overcast.
	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Sum		
	30.071	62.9	30.046	63.8	49.4	58.5	63.4	48.4	64.9	0.313		

Monthly Mean of the Barometer, corrected for Capillarity and reduced to 32° Fahr. ..... { 9 A.M. 29.995 | 3 P.M. 29.968 }

OBSERVANDA.

Height of the Cistern of the Barometer above a Fixed Mark on Waterloo Bridge ..... = 83 feet 2½ in.

..... above the mean level of the Sea (presumed about)..... = 95 feet.

The External Thermometer is 2 feet higher than the Barometer Cistern.

Height of the Receiver of the Rain Gauge above the Court of Somerset House ..... = 79 feet 0 in.

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The Barometer is divided into inches and decimals.

METEOROLOGICAL JOURNAL FOR JUNE, 1829.

1829. June.	9 o'clock, A.M. Barom.	9 o'clock, A.M. Attach. Therm.	3 o'clock, P.M. Barom.	3 o'clock, P.M. Attach. Therm.	Dew Point at 9 A.M. in degrees of Fahr.	External Thermometer. Fahrenheit. 9 A.M.	Fahrenheit. 3 P.M.	Self-registering. Lowest.	Self-registering. Highest.	Rain, in inches. Read off at 9 A.M.	Direction of the Wind at 9 A.M.	Remarks.
● ☽ 1	30.246	60.3	30.273	64.8	51	58.3	61.7	51.7	65.0		NNW	Cloudy—light wind.
♂ 2	30.329	69.6	30.280	67.8	51	64.7	71.6	50.4	72.3		NW	Fine—light clouds and wind.
☿ 3	30.236	69.3	30.175	71.3	58	71.3	77.1	60.2	77.8		WNW	Fine—light clouds. Rain at night.
♃ 4	30.152	74.2	30.075	72.3	50	66.8	71.7	60.6	73.2	0.011	NW	Fine—light clouds.
♀ 5	30.034	73.3	30.063	69.8	45	62.3	64.8	55.5	65.8		NNW	Fine and clear—light clouds and wind.
♄ 6	30.263	69.6	30.250	64.6	37	55.5	56.5	45.0	58.8		NNE	Fine and clear—cloudy—light wind.
⊙ 7	30.350	65.9	30.352	59.6	40	52.9	56.9	41.1	58.2		NNE	Fine & clear—light clouds—light wind.
☽ 8	30.333	59.5	30.336	62.1	51	55.9	58.9	49.1	61.3		NW	A.M. Thickly overcast—light rain. P.M. Fine.
♂ 9	30.337	57.8	30.307	62.3	48	54.5	61.7	45.9	62.6	0.016	NNE	Fine and clear—light wind.
☿ 10	30.394	63.7	30.368	64.2	47	60.3	65.7	46.3	66.8		N	Fine and clear—light wind.
♃ 11	30.426	62.3	30.355	63.8	47	55.3	66.2	46.8	68.5		E	Fine and clear—lightly cloudy—light wind.
♀ 12	30.355	62.2	30.308	65.4	54	58.7	69.0	51.4	69.8		ESE	Fine—light clouds.
♄ 13	30.341	73.6	30.234	69.5	55	69.2	74.6	53.7	76.6		SW	Fine—light clouds.
⊙ 14	30.248	70.3	30.175	71.3	60	68.6	76.5	58.7	78.3		SW	Fine and clear—light clouds.
☽ 15	30.115	67.9	30.045	71.0	55	65.3	72.7	56.3	74.2		WSW	A.M. High clouds. P.M. Clear and cloudless.
♂ 16	29.860	71.5	29.860	67.3	50	60.4	59.7	56.3	65.3		NW	Cloudy. Rain at noon.
○ ☿ 17	29.883	72.8	29.834	69.8	47	63.8	65.3	48.7	68.7	0.019	W	Fine and clear. Rain at night.
♃ 18	29.912	67.0	29.973	67.3	47	58.7	61.5	52.1	65.2	0.111	NNW	Clear—cloudy—light wind. Thunder P.M.
♀ 19	30.064	70.7	30.010	68.8	48	62.9	67.3	53.0	70.5		SW	Fine—lightly cloudy.
♄ 20	29.906	74.7	29.828	69.7	48	69.8	69.8	55.3	71.2		SE	Fine and clear—light clouds and wind. Rain at night.
⊙ 21	29.807	74.5	29.822	72.0	53	67.5	69.2	60.3	70.7	0.114	SE	Fine—interfused clouds—light breeze.
☽ 22	29.829	67.5	29.812	68.5	57	62.7	64.7	58.3	71.3	0.111	SE	Lowering—frequent showers.
♂ 23	29.945	75.2	29.963	72.3	55	69.7	72.7	57.3	74.3	0.047	S	Fine—light diffused clouds. Rain at night.
☿ 24	30.070	77.9	30.072	74.3	59	72.5	72.4	61.7	74.6	0.350	S	Fine—very clear—light wind.
♃ 25	30.104	76.9	30.017	75.7	56	72.3	71.3	58.2	74.7		ESE	A.M. Cloudless—light wind. P.M. Broken clouds.
♀ 26	29.963	75.1	29.910	74.8	59	69.5	71.8	60.6	74.6		W	Fine and clear—light clouds. Evening rainy.
♄ 27	29.654	70.8	29.545	74.0	60	66.4	70.4	60.3	73.3	0.153	SSE	Cloudy—light showers.
⊙ 28	29.454	70.5	29.490	72.8	62	66.7	70.4	61.5	72.3	0.156	SSE	Interfused clouds—light wind. Thunder P.M. Rain at night.
☽ 29	29.680	65.5	29.739	66.9	58	57.7	59.0	55.4	63.7	0.305	NNW	Cloudy and foggy—light rain.
♂ 30	29.766	66.8	29.750	70.2	53	63.7	67.3	52.6	69.3	0.008	WSW	A.M. Fine. P.M. Cloudy—rain.
	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Sum		
	30.069	69.2	30.041	68.8	52.0	63.5	67.3	54.1	69.6	1.401		

Monthly Mean of the Barometer, corrected for Capillarity and reduced to 32° Fahr. ..... { 9 A.M. 29.978 | 3 P.M. 29.950 }

OBSERVANDA.

Height of the Cistern of the Barometer above a Fixed Mark on Waterloo Bridge ..... = 83 feet 2½ in.

..... above the mean level of the Sea (presumed about) ..... = 95 feet.

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